Introductory Physics

A Mastery-Oriented Curriculum

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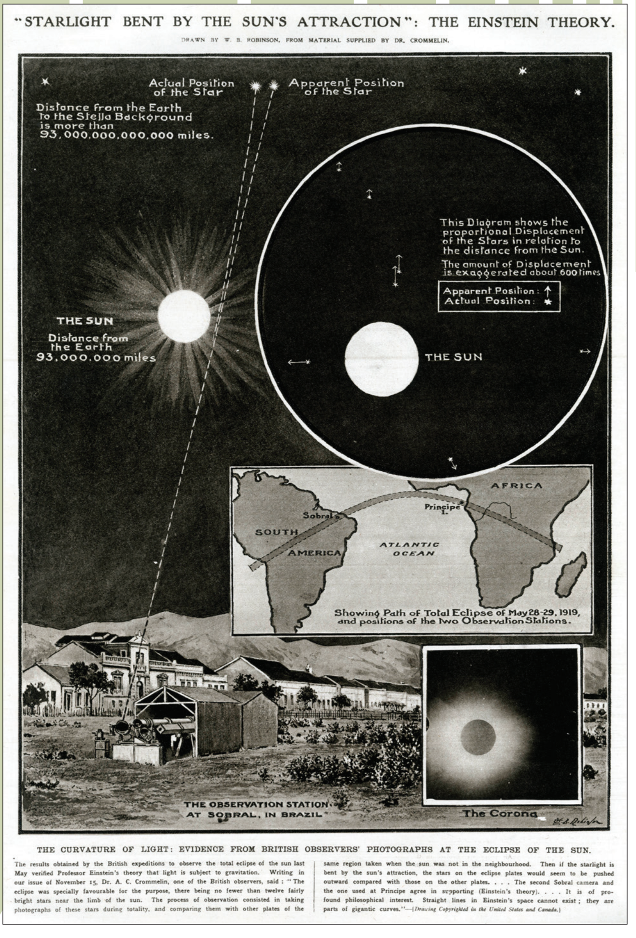
CHAPTER 1
The Nature of Scientific Knowledge


"STARLIGHT BENT BY THE SUN'S ATTRACTION": THE EINSTEIN THEORY.

DRAWN BY W. B. ROBINSON, FROM MATERIAL SUPPLIED BY DR. CROMMELIN.

THE CURVATURE OF LIGHT: EVIDENCE FROM BRITISH OBSERVERS' PHOTOGRAPHS AT THE ECLIPSE OF THE SUN.

The results obtained by the British expeditions to observe the total eclipse of the sun last May verified Professor Einstein's theory that light is subject to gravitation. Writing in our issue of November 15, Dr. A. C. Crommelin, one of the British observers, said: "The eclipse was specially favourable for the purpose, there being no fewer than twelve fairly bright stars near the limb of the sun. The process of observation consisted in taking photographs of these stars during totality, and comparing them with other plates of the same region taken when the sun was not in the neighbourhood. Then if the starlight is bent by the sun's attraction, the stars on the eclipse plates would seem to be pushed outward compared with those on the other plates. . . . The second Sobral camera and the one used at Principe agree in supporting (Einstein's theory). . . . It is of profound philosophical interest. Straight lines in Einstein's space cannot exist; they are parts of gigantic curves."—*[Drawing Copyrighted in the United States and Canada.]*

Theory → Hypothesis → Experiment

In 1915, Albert Einstein produced his general theory of relativity. In 1917, Einstein announced an amazing new hypothesis: according to the theory, light traveling through space bends as it passes near a star. In 1919, this hypothesis was confirmed by teams under the leadership of Sir Arthur Eddington, using photographs taken of stars positioned near the sun in the sky during a solar eclipse.

The image above appeared in the Illustrated London News.

OBJECTIVES

After studying this chapter and completing the exercises, students will be able to do each of the following tasks, using supporting terms and principles as necessary:

1. Define science, theory, hypothesis, and scientific fact.
2. Explain the difference between truth and scientific facts and describe how we obtain knowledge of each.
3. Describe the difference between General Revelation and Special Revelation and relate these to our definition of truth.
4. Describe the "Cycle of Scientific Enterprise," including the relationships between facts, theories, hypotheses, and experiments.
5. Explain what a theory is and describe the two main characteristics of a theory.
6. Explain what is meant by the statement, "a theory is a model."
7. Explain the role and importance of theories in scientific research.
8. State and describe the steps of the "scientific method."
9. Define explanatory, response, and lurking variables in the context of an experiment.
10. Explain why experiments are designed to test only one explanatory variable at a time. Use the procedures the class followed in the Pendulum Experiment as a case in point.
11. Explain the purpose of the control group in an experiment.
12. Describe the possible implications of a negative experimental result. In other words, if the hypothesis is not confirmed, explain what this might imply about the experiment, the hypothesis, or the theory itself.

1.1 Modeling Knowledge

1.1.1 Kinds of Knowledge

There are many different kinds of knowledge. One kind of knowledge is *truth*. As Christians, we are very concerned about truth because of its close relation to knowledge revealed to us by God. Scientific facts and theories constitute a different kind of knowledge, and as students of the natural sciences we are also concerned about these.

Some people handle the distinction between the truths of the faith and scientific knowledge by referring to religious teachings as one kind of truth and scientific teaching as a different kind of truth. The problem here is that there are not different kinds of truth. There is only *one* truth, but there *are* different kinds of knowledge. Truth is one kind of knowledge, and scientific knowledge is a different kind of knowledge.

We are going to unpack this further over the next few pages, but here is a taste of where we are going. Scientific knowledge is not static. It is always changing as new discoveries are made. On the other hand, the core teachings of Christianity do not change. They are always true. We know this because God reveals them to us in his Word, which is true. This difference between scientific knowledge and knowledge from Scripture indicates to us that the knowledge we have from the Scriptures is a different kind of knowledge than what we learn from scientific investigations.

I have developed a model of knowledge that emphasizes the differences between what God reveals to us and what scientific investigations teach us. This model is not perfect (no model is), nor is it exhaustive, but it is very useful, as all good models are. Our main goal in the next few sections is to develop this model of knowledge. The material in this chapter is crucial if you wish to have a proper understanding of what science is all about.

To understand science correctly, we need to understand what we mean by scientific knowledge. Unfortunately, there is much confusion among non-scientists about the nature of scientific knowledge and this confusion often leads to misunderstandings when we talk about scientific findings and scientific claims. This is nothing new. Misconceptions about scientific claims have plagued public discourse for thousands of years and continue to do so to this day. This confusion is a severe problem, one much written about within the scientific community in recent years.

To clear the air on this issue, it is necessary to examine what we mean by the term *truth*, as well as the different ways we discover truth. Then we must discuss the specific characteristics of scientific knowledge, including the key scientific terms *fact*, *theory*, and *hypothesis*.

1.1.2 What Is Truth and How Do We Know It?

Epistemology, one of the major branches of philosophy, is the study of what we can know and how we know it. Both philosophers and theologians claim to have important insights on the issue of knowing truth, and because of the roles science and religion have played in our culture over the centuries, we need to look at what both philosophers and theologians have to say. The issue we need to treat briefly here is captured in this question: What is truth and how do we know it? In other words, what do we mean when we say something is *true*? And if we can agree on a definition for truth, how can we *know* whether something is true?

These are really complex questions, and philosophers and theologians have been working on them for thousands of years. But a few simple principles will be adequate for our purpose.

As for what truth is, my simple but practical definition is this:

Truth is the way things really are.

Whatever reality is like, that is the truth. If there *really* is life on other planets, then it is true to say, "There is life on other planets." If you live in Poughkeepsie, then when you say, "I live in Poughkeepsie" you are speaking the truth.

The harder question is: How do we know the truth? According to most philosophers, there are two ways that we can know truth, and these involve either our senses or our use of reason. First, truths that are obvious to us just by looking around are said to be *evident*. It is evident that birds can fly. No proof is needed. So the proposition, "Birds can fly," conveys truth. Similarly, it is evident that humans can read books and that birds cannot. Of course, when we speak of people knowing truth this way we are referring to people whose perceptive faculties are functioning normally.

The second way philosophers say we can know truth is through the valid use of logic. Logical conclusions are typically derived from a sequence of logical statements called a *syllogism*, in which two or more statements (called *premises*) lead to a conclusion. For example, if we begin with the premises, "All men are mortal," and, "Socrates was a man," then it is a valid conclusion to state, "Socrates was mortal." The truth of the conclusion of a logical syllogism definitely depends on the truth of the premises. The truth of the conclusion also depends on the syllogism having a valid structure. Some logical structures are not logically valid. (These invalid structures are called *logical fallacies*.) If the premises are true and the structure is valid, then the conclusion must be true.

So the philosophers provide us with two ways of knowing truth that most people agree upon—truths can be evident (according to our senses) or they can be proven (by valid use of reason from true premises).

Believers in some faith traditions—including Christianity—argue for a crucial third possibility for knowing truth, which is by revelation from supernatural agents such as God or angels. Jesus said, "I am the way, and the truth, and the life" (John 14:6). As Christians, we believe that Jesus was "God with us" and that all he said and did were revelations of truth to us from God the Father. Further, we believe that the Bible is inspired by God and reveals truth to us. We return to the ways God reveals truth to us at the end of this section.

Obviously, not everyone accepts the possibility of knowing truth by revelation. Specifically, those who do not believe in God do not accept the possibility of revelations from God. Additionally, there are some who accept the existence of a transcendent power or being, but do not accept the possibility of revelations of truth from that power. So this third way of knowing truth is embraced by many people, but certainly not by everyone.

Few people would deny that knowing truth is important. This is why we started our study by briefly exploring what truth is. But this is a book about science, and we need now to move to addressing a different question: what does *science* have to do with truth? The question is not as simple as it seems, as evidenced by the continuous disputes between religious and scientific communities stretching back over the past 700 years. To get at the relationship between science and truth, we first look at the relationship between propositions and truth claims.

1.1.3 Propositions and Truth Claims

Not all that passes as valid knowledge can be regarded as *true*, which I defined in the previous section as "the way things really are." In many circumstances—maybe most—we do not actually *know* the way things really are. People do, of course, often use propositions or statements with the intention of conveying truth. But with other kinds of statements, people intend to convey something else.

Let's unpack this with a few example statements. Consider the following propositions:

1. I have two arms.
2. My wife and I have three children.
3. I worked out at the gym last week.
4. My car is at the repair shop.
5. Texas gained its independence from Mexico in 1836.
6. Atoms are composed of three fundamental particles—protons, neutrons, and electrons.
7. God made the world.

Among these seven statements are actually three different types of claims. From the discussion in the previous section you may already be able to spot two of them. But some of these statements do not fit into any of the categories we explored in our discussion of truth. We can discover some important aspects about these claims by examining them one by one. So suppose for a moment that I, the writer, am the person asserting each of these statements as we examine the nature of the claim in each case.

I have two arms. This is true. I do have two arms, as is evident to everyone who sees me.

My wife and I have three children. This is true. To me, it is just as evident as my two arms. I might also point out that it is true regardless of whether other people believe me when I say it. (Of course, someone could claim that I am delusional, but let's just keep it simple here and assume I am in normal possession of my faculties.) This bit about the statement being true regardless of others' acceptance of it comes up because of a slight difference here between the statement about children and the statement about arms. Anyone who looks at me will accept the truth that I have two arms. It will be evident, that is, obvious, to them. But the truth about my children is only really evident to a few people (my wife and I, and perhaps a few doctors and close family members). Nevertheless, the statement is true.

I worked out at the gym last week. This is also true; I did work out last week. The statement is evident to me because I clearly remember going there. Of course, people besides myself must depend on me to know it because they cannot know it directly for themselves unless they saw me there. Note that I cannot prove it is true. I can produce evidence, if needed, but the statement cannot be proven without appealing to premises that may or may not be true. Still, the statement is true.

My car is at the repair shop. Here is a statement that we cannot regard as a truth claim. It is merely a statement about where I understand my car to be at present, based on where I left it this morning and what the people at the shop told me they were going to do with it. For all I know, they may have taken my car joy riding and presently it may be flying along the back roads of the Texas hill country. I *can* say that the statement is correct so far as I know.

Texas gained its independence from Mexico in 1836. We Texans were all taught this in school and we believe it to be correct, but as with the previous statement we must stop short of calling this a truth claim. It is certainly a *historical fact*, based on a lot of historical evidence. The statement is correct so far as we know. But it is possible there is more to that story than we know at present (or will ever know) and none of those now living were there.

Atoms are composed of three fundamental particles—protons, neutrons, and electrons. This statement is, of course, a scientific fact. But like the previous two statements, this statement is not—surprise!—a truth claim. We simply do not know the truth about atoms. The truth about atoms is clearly not evident to our senses. We cannot guarantee the truth of any premises we might use to construct a logical proof about the insides of atoms, so proof is not able to lead us to the truth. And so far as I know, there are no supernatural agents who have revealed to us anything about atoms. So we have no access to knowing how atoms really are. What we do have are the data from many experiments, which may or may not tell the whole story. Atoms may have other components we don't know about yet. The best we can say about this statement is that *it is correct so far as we know* (that is, so far as the scientific community knows).

God made the world. This statement clearly is a truth claim, and we Christians joyfully believe it. But other people disagree on whether the statement is true. I include this example here because we soon see what happens when scientific claims and religious truth claims get confused. I hope you are a Christian, but regardless of whether you are, the issue is important. We all need to learn to speak correctly about the different claims people make.

To summarize this section, some statements we make are evidently or obviously true. But for many statements, we must recognize that we don't know if they actually are true. The best we can say about these kinds of statements—and scientific facts are like this—is that they are correct so far as we know. Finally, there are metaphysical or religious statements about which people disagree; some claim they are true, some deny the same, and some say there is no way to know.

1.1.4 Truth and Scientific Claims

Let's think a bit further about the truth of reality, both natural and supernatural. Most people agree that regardless of what different people think about God and nature, there is some actual truth or *reality* about nature and the supernatural. Regarding nature, there is some full reality about the way, say, atoms are structured, regardless of whether we currently understand that structure correctly. So far as we know, this reality does not shift or change from day to day, at least not since the early history of the universe. So the reality about atoms—the truth about atoms—does not change.

And regarding the supernatural, there is some reality about the supernatural realm, regardless of whether anyone knows what that is. Whatever these realities are, they are *truths*, and these truths do not change either.

Now, I have observed over the years that since (roughly) the beginning of the 20th century, careful scientists do not refer to scientific claims as truth claims. They do not profess to knowing the ultimate truth about how nature *really* is. For example, Niels Bohr, one of the great physicists of the 20th century, said, "It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can *say* about nature." Scientific claims are understood to be statements about *our best understanding* of the way things are. Most scientists believe that over time our scientific theories get closer and closer to the truth of the way things really are. But when they are speaking carefully, scientists do not claim that our present understanding of this or that is the truth about this or that.

1.1.5 Truth vs. Scientific Facts

Whatever the truth is about the way things are, that truth is presumably absolute and unchanging. If there is a God, then that's the way it is, period. And if matter is made of atoms as we think it is, then that is the truth about matter and it is always the truth. But what we call scientific facts, by their very nature, are not like this. Scientific facts are subject to change, and sometimes do, as new information comes becomes known through ongoing scientific research. Our definitions for truth and for scientific facts need to take this difference into account. As we have seen, truth is the way things really are. By contrast, here is a definition for *scientific facts*:

> A scientific fact is a proposition that is supported by a great deal of evidence.
>
> Scientific facts are discovered by observation and experiment, and by making inferences from what we observe or from the results of our experiments.
>
> A scientific fact is *correct so far as we know*, but can change as new information becomes known.

So scientific facts can change. Scientists do not put them forward as truth claims, but as propositions that are correct so far as we know. In other words, scientific facts are *provisional*. They are always subject to revision in the future. As scientists make new scientific

Examples of Changing Scientific Facts

In 2006, the planet Pluto was declared not to be a planet any more.

In the 17th century, the fact that the planets and moon all orbit the earth changed to the present fact that the planets all orbit the sun, and only the moon orbits the earth.

At present we know of only one kind of matter that causes gravitational fields. This is the matter made up of protons, electrons, and neutrons, which we discuss in a later chapter. But scientists now think there may be another kind of matter contributing to the gravitational forces in the universe. They call it "dark matter" because apparently this kind of matter does not reflect or refract light the way ordinary matter does. (We also study reflection and refraction later on.) For the existence of dark matter to become a scientific fact, a lot of evidence is required, evidence which is just beginning to emerge. If we are able to get enough evidence, then the facts about matter will change.

discoveries, they must sometimes revise facts that were formerly considered to be correct. But the truth about reality, whatever it is, is absolute and unchanging.

The distinction between truth and scientific facts is crucial for a correct understanding of the nature of scientific knowledge. Scientific facts can change; truth does not.

1.1.6 Revelation of Truth

In Section 1.1.2, we examined the ways we can know truth. Here we need to say a bit more about what Christian theology says about revealed truth.

Christians believe that the supreme revelation of God to us was through Jesus Christ in the incarnation. Those who knew Jesus and those who heard Jesus teach were receiving direct revelation from God. Jesus said, "Whoever has seen me has seen the Father" (John 14:9).

Jesus no longer walks with us on the earth in a physical body (although we look forward to his return when he will again be with us). But Christians believe that when Jesus departed he sent his Holy Spirit to us, and today the Spirit guides us in the truth. According to traditional Christian theology, God continues to reveal truth to us through the Spirit in two ways: *Special Revelation* and *General Revelation*. Special Revelation is the term theologians use to describe truths God teaches us in the Bible, his Holy Word. General Revelation refers to truths God teaches us through the world he made. Sometimes theologians have described Special and General Revelation as the two "books" of God's revelation to us, the book of God's *Word* (the Bible) and the book of God's *Works* (nature). And it is crucial to note that the truths revealed in God's Word and those revealed in his Works *do not conflict*.

Truth is not discovered the same way scientific facts are. Truth is true for all people, all times, and all places. Truth never changes. Here are just a few examples of the many truths revealed in God's Word:

- Jesus is the divine Son of God (Matthew 16:16).
- All have sinned and fall short of what God requires (Romans 3:23).
- All people must die once and then face judgment (Hebrews 9:27).
- God is the creator of all that is (Colossians 1:16, Revelation 4:11).
- God loves us (John 3:16).

Each of these statements is true, and we know they are true because God has revealed them to us in his word. (The reasons for believing God's word are important for all of us to know and understand, but that is a subject for a different course of study.)

1.2 The Cycle of Scientific Enterprise

1.2.1 Science

Having established some basic principles about the distinction between scientific facts and truth, we are now ready to define *science* itself and examine what science is and how it works. Here is a definition:

> Science is the process of using experiment, observation, and logical thinking to build "mental models" of the natural world. These mental models are called *theories*.

We do not and cannot know the natural world perfectly or completely, so we construct models of how it works. We explain these models to one another with descriptions, diagrams, and mathematics. These models are our scientific theories. Theories never explain the world to us perfectly. To know the world perfectly, we would have to know the absolute truth about reality just as God knows it, which in this present age we do not. So theories always have their limits, but we hope they become more accurate and more complete over time, accounting for more and more physical phenomena (data, scientific facts), and helping us to understand the natural world as a coherent whole.

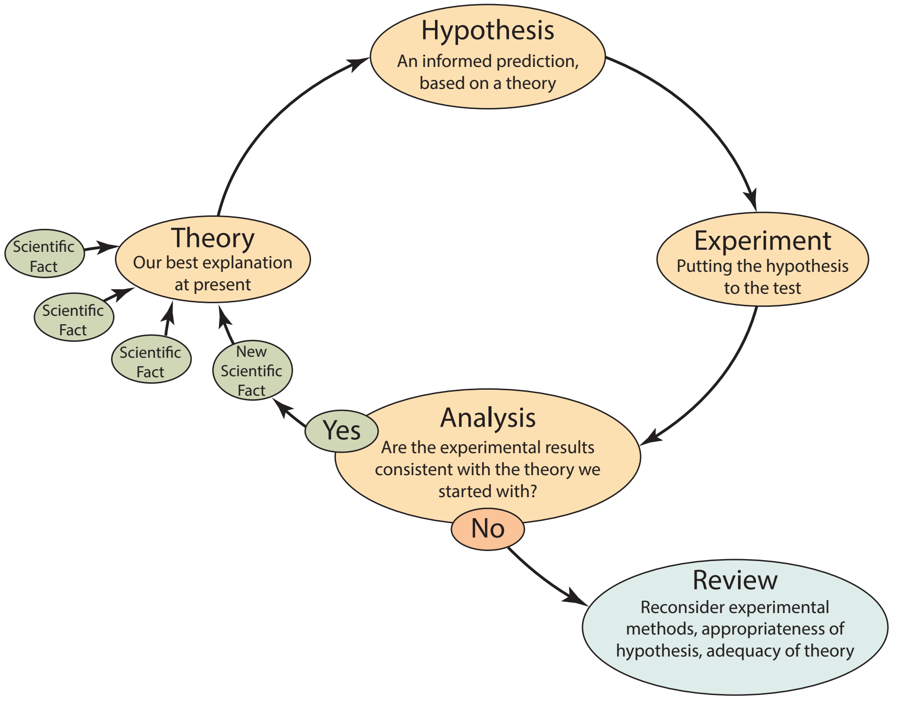
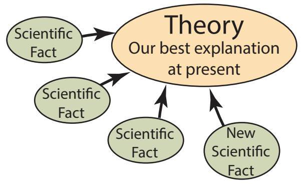


Figure 1.1. The Cycle of Scientific Enterprise.

Scientific knowledge is continuously changing and advancing through a cyclic process that I call the *Cycle of Scientific Enterprise*, represented in Figure 1.1. In the next few sections, we examine the individual parts of this cycle in detail.

1.2.2 Theories

Theories are the grandest thing in science. In fact, it is fair to say that theories are the *glory* of science, and developing good theories is what science is all about. Electromagnetic field theory, atomic theory, quantum theory, the general theory of relativity—these are all theories in physics that have had a profound effect on scientific progress and on the way we all live.[1]



Now, even though many people do not realize it, *all scientific knowledge is theoretically based.* Let me explain. A *theory* is a mental model or explanatory system that explains and relates together most or all of the scientific facts (the data) in a certain sphere of knowledge. A theory is not a hunch or a guess or a wild idea. Theories are the mental structures we use to make sense of the data we have. We cannot understand any scientific data without a theory to organize it and explain it. This is why I write that all scientific knowledge is theoretically based. And for this reason, it is inappropriate and scientifically incorrect to scorn these explanatory systems as "merely a theory" or "just a theory." Theories are explanations that account for a lot of different scientific facts. If a theory has stood the test of time, that means it has wide support within the scientific community.

It is popular in some circles to speak dismissively of certain scientific theories, as if they represent some kind of untested speculation. It is simply incorrect—and very unhelpful—to speak this way. As students in high school science, one of the important things you need to understand is the nature of scientific knowledge, the purpose of theories, and the way scientific knowledge progresses. These are the issues this chapter is about.

All useful scientific theories must possess several characteristics. The two most important ones are:

- The theory accounts for and explains most or all of the related scientific facts.
- The theory enables new hypotheses to be formed and tested.

Theories typically take decades or even centuries to gain credibility. If a theory gets replaced by a new, better theory, this also usually takes decades or even centuries to happen. No theory is ever "proven" or "disproven" and we should not speak of them in this way. We also should not speak of them as being "true" because, as we have seen, we do not use the word "truth" when speaking of scientific knowledge. Instead, we speak of scientific facts being correct so far as we know, or of current theories as representing our best understanding, or of theories being successful and useful models that lead to accurate predictions.

An experiment in which the hypothesis is supported by the experimental result is said to support the theory. After such an experiment, the theory is stronger but it is not proven. If a hypothesis is not supported by an experiment, the theory might be weakened but it is not disproven. Scientists require a great deal of experimental evidence before a new theory can be established as the best explanation for a body of data. This is why it takes so long for theories to become widely accepted. And since no theory ever explains everything perfectly,

1 The term *law* is just a historical (and obsolete) term for what we now call a theory.

there are always phenomena we know about that our best theories do not adequately explain. Of course, scientists continue their work in a certain field hoping eventually to have a theory that does explain all the scientific facts. But since no theory explains everything perfectly, it is impossible for one experimental failure to bring down a theory. Just as it takes a lot of evidence to establish a theory, so it takes a large and growing body of conflicting evidence before scientists abandon an established theory.

At the beginning of this section, I state that theories are mental *models*. This statement needs a bit more explanation. A model is a representation of something, and models are designed for a purpose. You have probably seen a model of the organs in the human body in a science classroom or textbook. A model like this is a physical model and its purpose is to help people understand how the human body is put together. A mental model is not physical; it is an intellectual understanding, although we often use illustrations or physical models to help communicate to one another our mental ideas. But as in the example of the model of the human body, a theory is also a model. That is, a theory is a representation of how part of the world works. Frequently, our models take the form of mathematical equations that allow us to make numerical predictions and calculate the results of experiments. The more accurately a theory represents the way the world works, which we judge by forming new hypotheses and testing them with experiments, the better and more successful the theory is.

To summarize, a successful theory represents the natural world accurately. This means the model (theory) is useful because if a theory is an accurate representation, then it leads

Examples of Famous Theories

In the next chapter, we encounter Einstein's general theory of relativity, one of the most important theories in modern physics. Einstein's theory represents our best current understanding of how gravity works.

Another famous theory we address later is the kinetic theory of gases, our present understanding of how molecules of gas too small to see are able to create pressure inside a container.

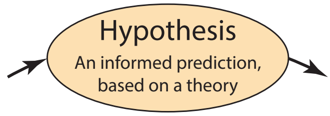
Key Points About Theories

1. A theory is a way of modeling nature, enabling us to explain why things happen in the natural world from a scientific point of view.
2. A theory tries to account for and explain the known facts that relate to it.
3. Theories must enable us to make new predictions about the natural world so we can learn new facts.
4. Strong, successful theories are the glory and goal of scientific research.
5. A theory becomes stronger by producing successful predictions that are supported by experiment. A theory is gradually weakened when new experimental results repeatedly turn out to be inconsistent with the theory.
6. It is incorrect to speak dismissively of successful theories because theories are not just guesses.
7. We don't speak of theories as being proven or disproven. Instead, we speak of them in terms such as how successful they have been at making predictions and how accurate the predictions have been.

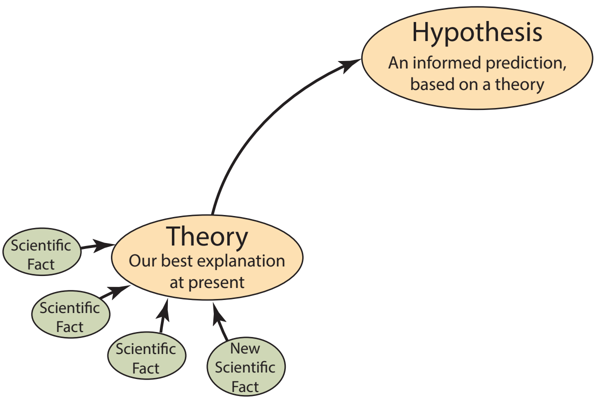
Figure 1.2. Key points about theories.

to accurate predictions about nature. When a theory repeatedly leads to predictions that are confirmed in scientific experiments, it is a strong, useful theory. The key points about theories are summarized in Figure 1.2.

1.2.3 Hypotheses



A *hypothesis* is a positively stated, informed prediction, with an explanation, about what will happen in certain circumstances. We say a hypothesis is an *informed* prediction because when we form hypotheses we are not just speculating out of the blue. We are applying a certain theoretical understanding of the subject to the new situation before us and predicting what will happen or what we expect to find in the new situation based on the theory the hypothesis is coming from. Every scientific hypothesis is based on a particular theory.



Often hypotheses are worded as *if-then-because* statements, such as, "If various forces are applied to a pickup truck, then the truck accelerates at a rate that is in direct proportion to the net force because of Newton's second law." Every scientific hypothesis is based on a theory and it is the hypothesis that is directly tested by an experiment. If the experiment turns out the way the hypothesis predicts, the hypothesis is supported by the experimental result and the theory it came from is strengthened. Of course, the hypothesis may not

Key Points About Hypotheses

1. A hypothesis is an informed prediction about what will happen in certain circumstances.
2. Every hypothesis is based on a particular theory.
3. Well-formed scientific hypotheses must be testable, which is what scientific experiments are designed to do.

Figure 1.3. Key points about hypotheses.

Examples of Famous Hypotheses

Einstein used his general theory of relativity to make an incredible prediction in 1917: that gravity causes light to bend as it travels through space. In the next chapter, you read about the stunning result that occurred when this hypothesis was put to the test.

The year 2012 was a very important year for the standard theory in the world of subatomic particles, called the Standard Model. In 1964, this theory led to the prediction that there are weird particles in nature, now called Higgs Bosons, which no one had ever detected. Until 2012, that is! An enormous machine that could detect these particles, called the Large Hadron Collider, was built in Switzerland and completed in 2008. In 2012, scientists announced that the Higgs Boson had been detected at last, a major victory for the Standard Model, and for Peter Higgs, the physicist who first proposed the particle that now bears his name.

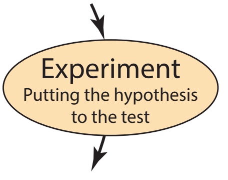
be supported by the experiment. We see how scientists respond to this situation in Section 1.2.6.

The terms *theory* and *hypothesis* are often used interchangeably in common speech, but in science they mean different things. For this reason you should make note of the distinction.

One more point about hypotheses. A hypothesis that cannot be tested is not a scientific hypothesis. For example, horoscopes purport to predict the future with statements like, "You will meet someone important to your career in the coming weeks." Statements like this are so vague they are untestable and do not qualify as scientific hypotheses.

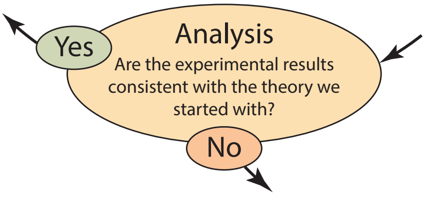
The key points about hypotheses are summarized in Figure 1.3.

1.2.4 Experiments



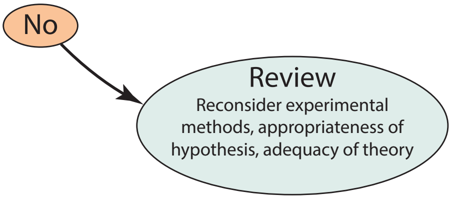
Experiments are tests of the predictions in hypotheses, under controlled conditions. Effective experiments are difficult to perform. Thus, for any experimental outcome to become regarded as a scientific fact it must be replicated by several different experimental teams, often working in different labs around the world. Scientists have developed rigorous methods for conducting valid experiments. We consider these briefly in Section 1.3.

1.2.5 Analysis



In the Analysis phase of the Cycle of Scientific Enterprise, researchers must interpret the experimental results. The results of an experiment are essentially data, and data always have to be interpreted. The main goal of this analysis is to determine whether the experimental data support the hypothesis. If they do, then the experiment has produced new scientific facts that are consistent with the original theory because the hypothesis is based on that theory. As a result, the support for the theory has increased—the theory was successful in generating a hypothesis that was supported by experiment. As a result of the experiment, our confidence in the theory as a useful model has increased and the theory is even more strongly supported than before.

1.2.6 Review



If the outcome of an experiment does not support the hypothesis, the researchers must consider all the possibilities for why this might have happened. Why didn't our theory, which is our best explanation of how things work, enable us to form a correct prediction? There are a number of possibilities, beginning with the experiment and going backwards around the cycle:

- The experiment may have been flawed. Scientists double check everything about the experiment, making sure all equipment is working properly, double checking the calculations, looking for unknown factors that may have inadvertently influenced the outcome, verifying that the measurement instruments are accurate enough and precise enough to do the job, and so on. They also wait for other experimental teams to try the experiment to see if they get the same results or different results, and then compare. (Although, naturally, every scientific team likes to be the first one to complete an important new experiment.)

- The hypothesis may have been based on a incorrect understanding of the theory. Maybe the experimenters did not understand the theory well enough, and maybe the hypothesis is not a correct statement of what the theory says will happen.
- The values used in the calculation of the hypothesis' predictions may not have been accurate or precise enough, throwing off the hypothesis' predictions.
- Finally, if all else fails, and the hypothesis still is not supported by experiment, it is time to look again at the theory. Maybe the theory can be altered to account for this new scientific fact. If the theory simply cannot account for the new scientific fact, then the theory has a weakness, namely, there are scientific facts it doesn't adequately account for. If enough of these weaknesses accumulate, then over a long period of time (like decades) the theory might eventually need to be replaced with a different theory, that is, another, better theory that does a better job of explaining all the scientific facts we know. Of course, for this to happen someone would have to conceive of a new theory, which usually takes a great deal of scientific insight. And remember, it is also possible that the scientific facts themselves can change.

1.3 The Scientific Method

1.3.1 Conducting Reliable Experiments

The so-called *scientific method* that you have been studying ever since about fourth grade is simply a way of conducting reliable experiments. Experiments are an important part of the *Cycle of Scientific Enterprise*, and so the scientific method is important to know. You probably remember studying the steps in the scientific method from prior courses, so they are listed in Table 1.1 without further comment.

We will be discussing variables and measurements a lot in this course, so we should take the opportunity here to identify some of the language researchers use during the experimental process. In a scientific experiment, the researchers have a question they are trying to answer (from the State the Problem step in the scientific method), and typically it is some kind of question about the way one physical quantity affects another one. So the researchers design an experiment in which one quantity can be manipulated (that is, deliberately varied in a controlled fashion) while the value of another quantity is monitored.

A simple example of this in everyday life that you can easily relate to is varying the amount of time you spend each week studying for your math class in order to see what effect the time spent has on the grades you earn. If you reduce the time you spend, will your grades go down? If you increase the time, will they go up? A precise answer depends on a lot of things, of course, including the person involved, but in general we would all agree that if a stu-

The Scientific Method	
1. State the problem.	5. Collect data.
2. Research the problem.	6. Analyze the data.
3. Form a hypothesis.	7. Form a conclusion.
4. Conduct an experiment.	8. Repeat the work.

Table 1.1. Steps in the scientific method.

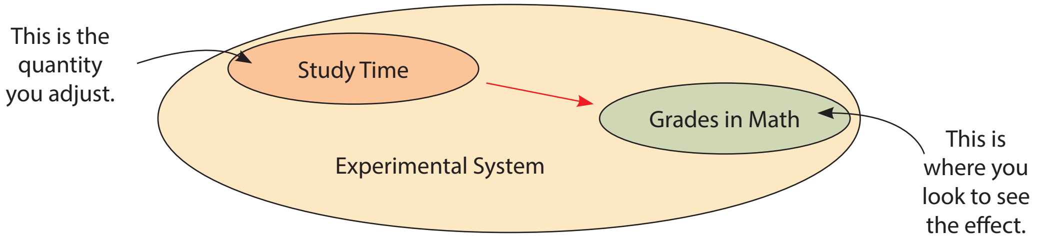


Figure 1.4. Study time and math grades in a simple experimental system.

dent varies the study time enough we would expect to see the grades vary as well. And in particular, we would expect more study time to result in higher grades. The way your study time and math grades relate together can be represented in a diagram such as Figure 1.4.

Now let us consider this same concept in the context of scientific experiments. An experiment typically involves some kind of complex system that the scientists are modeling. The system could be virtually anything in the natural world—a galaxy, a system of atoms, a mixture of chemicals, a protein, or a badger. The variables in the scientists' mathematical models of the system correspond to the physical quantities that can be manipulated or measured in the system. As I describe the different kinds of variables, refer to Figure 1.5.

1.3.2 Experimental Variables

When performing an experiment, the variable that is deliberately manipulated by the researchers is called the *explanatory variable*. As the explanatory variable is manipulated, the researchers monitor the effect this variation has on the *response variable*. In the example of study time versus math grade, the study time is the explanatory variable and the grade earned is the response variable.

Usually, a good experimental design allows only one explanatory variable to be manipulated at a time so that the researchers can tell definitively what its effect is on the response variable. If more than one explanatory variable were changing during the course of the experiment, researchers may not be able to tell which one was causing the effect on the response variable.

A third kind of variable that plays a role in experiments is the *lurking variable*. A lurking variable is a variable that affects the response variable without the researchers being aware of it. This is undesirable, of course, because with unknown influences present the researchers may not be able to make a correct conclusion about the effect of the known explanatory variables on the response variable under study. So researchers have to study

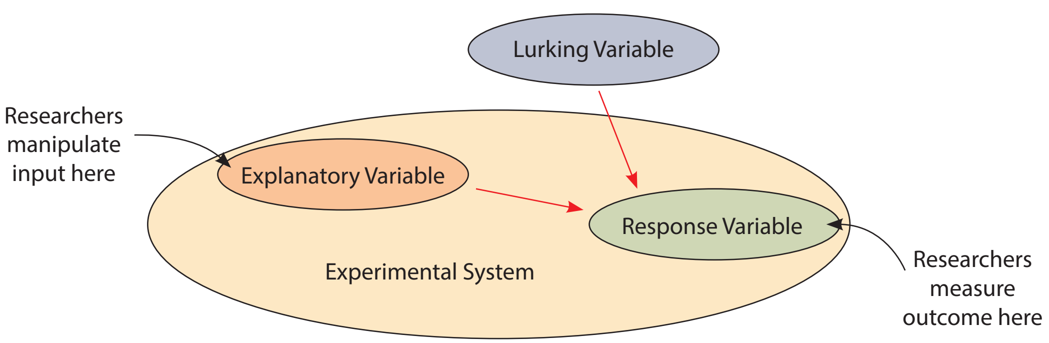


Figure 1.5. The variables in an experimental system.

their experimental projects very carefully to minimize the possibility of lurking variables affecting their results.

In our example about study time and math grades, there could be a number of lurking variables affecting the results of the experiment. Possible lurking variables include changes

Do You Know ... What are double-blind experiments?

The human mind is so powerful that if a person *believes* a new medication might help, the person's condition can sometimes improve even if the medication itself isn't doing a thing! This is amazing, but in medical research it means that the researchers can have a hard time determining whether a person is helped by the new medication, or by feeling positively about the medication, or even by the attention given to him or her by the doctor.

Pictured below is Lauren Wood, a clinician involved in vaccine research at the Center for Cancer Research, which is part of the National Cancer Institute. Just as with every other scientific researcher, Dr. Wood's research is conducted according to methods that have been developed to ensure that people's beliefs about the research don't influence the outcome of the research.

The approach is to divide the patients who will participate in testing a new medication into two groups, control and experimental. The experimental group is given the new medication. The control group is given a *placebo*—a fake medication such as a sugar pill—that has no effect on the person's medical condition. Further, none of the patients know whether they are given the placebo or the real medication. This technique, called a *blind experiment*, allows the researchers to determine whether a new medication actually helps, as they compare the results of the control and experimental groups.

But there's more. It turns out that *the researchers themselves* can affect the results of the experiment if they know which patients are receiving a placebo and which ones are receiving the medication under study. How can this happen? Well, if the researchers know who is getting the real medication, they might subconsciously act more positively with them than with other patients. This might be because the researchers expect those getting the new medication to improve, and this expectation gets subconsciously communicated to the patients. The positive attitude might be perceived as more encouraging and patients might improve just because of the encouragement!

The way around this dilemma is to use a *double-blind experiment*. In a double-blind experiment, neither the patients nor the researchers know which patients are getting the placebo and which are getting the real treatment. A team of technicians is in the middle, administering the medication and keeping records of who received what. The researchers are not allowed to see the lists until the research results are finalized. The double-blind experiment is the standard protocol followed today for new medical research.

in the difficulty of the material from one chapter to the next and variations in the student's ability to concentrate due to fatigue from seasonal sports activities.

1.3.3 Experimental Controls

The last thing we consider in this section is an important way researchers control an experiment to ensure the results are valid. You are probably aware that developing new medical treatments is one of the major goals of experimental research in the 21st century. Many experiments in the field of medical research are designed to test some new kind of treatment by comparing the results of the new treatment to those obtained using a conventional treatment or no treatment at all. This is the situation in medical research all the time for experiments testing new therapies, medications, or procedures.

Clinical trials are experiments conducted by researchers on people to test new therapies or medications. In experiments like these, the people (patients) involved in the study are divided into two groups—the *control group* and the *experimental group*. The control group receives no treatment or some kind of standard treatment. The experimental group receives the new treatment being tested. The results of the experimental group are assessed by comparing them to those of the control group.

Another example will help to clarify all these terms. Let's say researchers have developed a variety of fruit tree that they believe is more resistant to drought than other varieties. According to the researchers' *theoretical understanding* of how chemical reactions and water storage work in the biological systems of the plant, they *hypothesize* that the new variety of tree will be able to bear better fruit during drought conditions. To test this hypothesis by *experiment*, the scientists develop a group of the new trees. Then they place the trees in a test plot, along with other trees of other varieties, and see how they perform. Figure 1.6 shows a researcher working in an agricultural test plot. In our fruit tree example, the trees of the new variety are in the *experimental group* and the trees of the other varieties are in the *control group*.

The *response variable* is the quality of the plant's fruit. Researchers expect that under drought conditions the fruit of the new variety will be better than the fruit of the other varieties. The *explanatory variable* is the unique feature of the new variety that relates to the plant's use of water. The trees are exposed to drought conditions in the experiment. If the new variety produces higher quality fruit than the control group, then the hypothesis is supported, and the theory that led to the hypothesis has gained credibility through this success. One can imagine many different *lurking variables* that could affect the outcome of this experiment without the scientists' awareness. For example, the new variety trees could be planted in locations that receive different amounts of moisture or sun than the locations where the control group trees are, or, the nutrients in the soil in different locations might vary.

Figure 1.6. An agricultural research assistant working in a test plot.

In a good experimental design, researchers seek to identify such factors and take measures to ensure that they do not affect the outcome of the experiment. They do this by making sure there are trees from both the experimental group and the control group in all the different conditions the trees will experience. This way, variations in sunlight, soil type, soil water content, elevation, exposure to wind, and other factors will be experienced equally by trees in both groups.

Chapter 1 Exercises

As you go through the chapters in this book, always answer the questions in complete sentences, using correct grammar and spelling.

Here is a tip that will help improve the quality of your written responses: avoid pronouns! Pronouns almost always make your responses vague or ambiguous. If you want to receive full credit for written responses, avoid them. (Oops. I mean, avoid pronouns!)

Study Questions

Answer the following questions with a few complete sentences.

1. Distinguish between theories and hypotheses.
2. Explain why a single experiment can never prove or disprove a theory.
3. Explain how an experiment can still provide valuable data even if the hypothesis under test is not supported by the experimental result.
4. Explain the difference between truth and facts and describe the sources of each.
5. State the two primary characteristics of a theory.
6. Does a theory need to account for all known facts? Why or why not?
7. It is common to hear people say, "I don't accept that; it's just a theory." What is the error in a comment like this?
8. Distinguish between facts and theories.
9. Distinguish between explanatory variables, response variables, and lurking variables.
10. Why do good experiments that seek to test some kind of new treatment or therapy include a control group?
11. Explain specifically how the procedure you followed in the Pendulum Experiment satisfies every step of the "scientific method."
12. This chapter argues that scientific facts should not be regarded as true. Someone might question this and ask, If they aren't true, then what are they good for? Develop a response to this question.
13. Explain what a model is and why theories are often described as models.
14. Consider an experiment that does not deliver the result the experimenters had expected. In other words, the result is negative because the hypothesis is not supported by the data. There are many reasons why this might happen. Consider each of the following elements of the Cycle of Scientific Enterprise. For each one,

describe how it might be the driving factor that results in the experiment's failure to support the hypothesis.

a. the experiment
b. the hypothesis
c. the theory

15. Identify the explanatory and response variables in the Pendulum Experiment, and identify two realistic possibilities for ways the results may have been influenced by lurking variables.

Do You Know ... *How did Sir Humphry Davy become a hero?*

Sir Humphry Davy (1778–1829) was one of the leading experimenters and inventors in England in the early nineteenth century. He conducted many early experiments with gases; discovered sodium, potassium, and numerous other elements; and produced the first electric light from a carbon arc.

In the early nineteenth century, explosions in coal mines were frequent, resulting in much tragic loss of life. The explosions were caused by the miners' lamps igniting the methane gas found in the mines.

Davy became a national hero when he invented the Davy Safety Lamp (below). This lamp incorporated an iron mesh screen around the flame. The cooling from the iron reduces the flame temperature so the flame does not pass through the mesh, and thus cannot cause an explosion. The Davy Lamp was produced in 1816 and was soon in wide use.

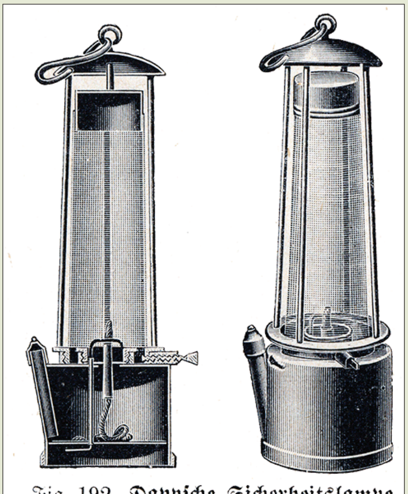


Fig. 192. Davysche Sicherheitslampe

Davy's experimental work proceeded by reasoning from first principles (theory) to hypothesis and experiment. Davy stated, "The gratification of the love of knowledge is delightful to every refined mind; but a much higher motive is offered in indulging it, when that knowledge is felt to be practical power, and when that power may be applied to lessen the miseries or increase the comfort of our fellow-creatures."

CHAPTER 2
Motion

Orrery

Orreries, mechanical models of the solar system, were well-known teaching tools in the 18th century, often forming the centerpiece of lessons on astronomy. They demonstrated Copernicus' theory that the earth and other planets orbit the sun. This example, from around 1750, is smaller but otherwise similar to George II's grand orrery.

This photo of the orrery was taken in the British Museum in London.

OBJECTIVES

Memorize and learn how to use these equations:

$$v = \frac{d}{t} \qquad a = \frac{v_f - v_i}{t}$$

After studying this chapter and completing the exercises, students will be able to do each of the following tasks, using supporting terms and principles as necessary:

1. Define and distinguish between velocity and acceleration.
2. Use scientific notation correctly with a scientific calculator.
3. Calculate distance, velocity, and acceleration using the correct equations, MKS and USCS units, unit conversions, and units of measure.
4. Use from memory the conversion factors, metric prefixes, and physical constants listed in Appendix A.
5. Explain the difference between accuracy and precision and apply these terms to questions about measurement.
6. Demonstrate correct understanding of precision by using the correct number of significant digits in calculations and rounding.
7. Describe the key features of the Ptolemaic model of the heavens, including all the spheres and regions in the model.
8. State several additional features of the medieval model of the heavens and relate them to the theological views of the Christian authorities opposing Copernicanism.
9. Briefly describe the roles and major scientific models or discoveries of Copernicus, Tycho, Kepler, and Galileo in the Copernican Revolution. Also, describe the significant later contributions of Isaac Newton and Albert Einstein to our theories of motion and gravity.
10. Describe the theoretical shift that occurred in the Copernican Revolution and how Christian officials (both supporters and opponents) were involved.
11. State Kepler's first law of planetary motion.
12. Describe how the gravitational theories of Kepler, Newton, and Einstein illustrate the way the Cycle of Scientific Enterprise works.

2.1 Computations in Physics

In this chapter, you begin mastering the skill of applying mathematics to the study of physics. To do this well, you must know a number of things about the way measurements are handled in scientific work. You also need to have a solid problem-solving strategy you can depend on to help you solve problems correctly without becoming confused. These topics are addressed in this chapter.

2.1.1 The Metric System

Units of measure are crucial in science. Science is about making measurements and a measurement without its units of measure is a meaningless number. For this reason, your answers to computations in scientific calculations must *always* show the units of measure.

The two major unit systems you should know about are the SI (from the French *Système international d'unités*), typically known in the United States as the metric system, and

Unit	Symbol	Quantity
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	electric current
kelvin	K	temperature
candela	Cd	luminous intensity
mole	mol	amount of substance

Table 2.1. The seven base units in the SI unit system.

the USCS (U.S. Customary System). You have probably studied these systems before and should already be familiar with some of the SI units and prefixes, so our treatment here will be brief.

If you think about it, you would probably agree that the USCS is cumbersome. One problem is that there are many different units of measure for every kind of physical quantity. For example, just for measuring length or distance we have the inch, foot, yard, and mile. The USCS is also full of random numbers like 3, 12, and 5,280, and there is no inherent connection between units for different types of quantities.

By contrast, the SI system is simple and has many advantages. There is usually only one basic unit for each kind of quantity, such as the meter for measuring length. Instead of having many unrelated units of measure for measuring quantities of different sizes, fractional and multiple prefixes based on powers of ten are used with the units to accommodate various sizes of measurements.

A second advantage is that since quantities with different prefixes are related by some power of ten, unit conversions can often be performed mentally. To convert 4,555 ounces into gallons, we first have to look up the conversion from ounces to gallons (which is hard to remember), and then use a calculator to perform the conversion. But to convert 40,555 cubic centimeters into cubic meters is simple—simply divide by 1,000,000 and you have 0.040555 m^3. (If you are not clear on the reason for dividing by 1,000,000, just hold on until we get to the end of Section 2.1.3.)

Another SI advantage is that the units for different types of quantities relate to one another in some way. Unlike the gallon and the foot, which have nothing to do with each other, the liter (a volume) relates to the centimeter (a length): 1 liter = 1,000 cubic centimeters.[1] For all these reasons, the USCS is not used much in scientific work. The SI system is the international standard and it is important to know it well.

In the SI unit system, there are seven *base units*, listed in Table 2.1. (In this text, we use only the first five of them.) There are also many additional units of measure, known as *derived units*. All the derived units are formed by various combinations of base units. To illustrate, below are a few examples of derived units that we discuss and use in this book. Note, however, that we won't be working much with the messy fractions; they are simply shown to illustrate how base units are combined to form derived units.

- the newton (N) is the SI unit for measuring force: $1\ \text{N} = 1\ \dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}$
- the joule (J) is the SI unit for measuring energy: $1\ \text{J} = 1\ \dfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$
- the watt (W) is the SI unit for measuring power: $1\ \text{W} = 1\ \dfrac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$

1 The liter is not actually an official SI unit of measure, but it is used all the time anyway in scientific work.

Using the SI system requires knowing the units of measure—base and derived—and the prefixes that are applied to the units to form fractional units (such as the centimeter) and multiple units (such as the kilometer). The complete list of metric prefixes is shown in Appendix A in Table A.1. The short list of prefixes you need to know by memory for use in this course is in Table A.2. Note that even though the kilogram is a base unit, prefixes are not added to the kilogram. Instead, prefixes are added to the gram to form units such as the milligram and microgram.

2.1.2 MKS Units

A handy subset of the SI system is the so-called *MKS system*. The MKS system uses only base units—such as the *meter*, *kilogram*, and *second* (hence, "MKS") as units for mass, length, and time—along with other units derived from the base units. The mass, length, and time units, and the symbols and variables used with them, are listed in Table 2.2.

Variable	Variable Symbol	Unit	Unit Symbol
length	*d* (distance) *L* (length) *h* (height) *r* (radius), etc.	meter	m
mass	*m*	kilogram	kg
time	*t*	second	s

Table 2.2. The three base units in the MKS system.

Dealing with different systems of units can become quite confusing. But the wonderful thing about sticking to the MKS system is that *any calculation performed with MKS units produces a result in MKS units*. This is why the MKS system is so handy. The MKS system dominates calculations in physics and we use it almost all the time in this course.

To convert the units of measure given in problems into MKS units, you must know the conversion factors listed in Appendix A in Tables A.2, A.3, and A.4. Table A.5 lists several common unit conversions that you are not required to memorize but should have handy when working problem assignments.

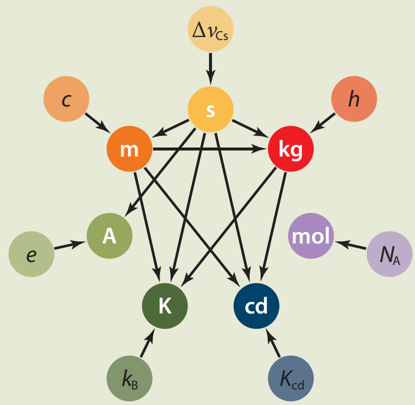
2.1.3 Converting Units of Measure

One of the most basic skills scientists and engineers use is re-expressing quantities into equivalent quantities with different units of measure. These calculations are called *unit conversions*. Mastery of this skill is essential for any student in high school science, and you use it *a lot* in this course. You have studied unit conversions in your math classes for the past few years. But this skill is so important in science that we are going take the time in this section to review in detail how to perform unit conversions.

Let's begin with the basic principle of how this works. First, you know that multiplying any value by unity (one) leaves its value unchanged. Second, you also know that in any fraction if the numerator and denominator are equivalent, the value of the fraction is *unity*, which means *one*. A "conversion factor" is simply a fractional expression in which the numerator and denominator are equivalent ways of writing the same physical quantity. This means a conversion factor is just a special way of writing unity (one). Third, we know that when multiplying fractions, factors that appear in both the numerator and denominator may be "cancelled out." So when performing common unit conversions, what we are doing is repeatedly multiplying our given quantity by unity so that cancellations alter the units of measure until they are expressed the way we wish. Since all we are doing is multiplying by one, the value of our original quantity is unchanged; it simply looks different because it is expressed with different units of measure.

Do You Know ... How are the base units defined?

The definitions of the base units all have interesting stories behind them. In the past, several units were defined by physical objects, such as a metal bar (the meter) or metal cylinder (the kilogram). But over time these definitions have been replaced. (The last one was the kilogram, replaced in 2019.) Now, each base unit is defined in terms of a physical constant that itself is defined with a specific, exact value. The definitions of all but two of the units also depend on other unit definitions, as the arrows in the graphic indicate. The official definition of the second is based on waves of light emitted by cesium atoms. The speed of light is defined as 299,792,458 m/s, and the meter is defined as the distance light travels in 1/299,792,458 seconds.



Let me elaborate a bit more on the idea of unity I mention above, using one common conversion factor as an example. School kids all learn that there are 5,280 feet in one mile, which means 5,280 ft = 1 mi. One mile and 5,280 feet are equivalent ways of writing the same length. If we place these two expressions into a fraction, the numerator and denominator are equivalent, so the value of the fraction is unity, regardless of the way we write it. The equation 5,280 ft = 1 mi can be written in a conversion factor two different ways, and the fraction equals unity either way:

$$\frac{5280\text{ ft}}{1\text{ mi}} = \frac{1\text{ mi}}{5280\text{ ft}} = 1$$

So if you have a measurement such as 43,000 feet that you wish to re-express in miles, the conversion calculation is written this way:

$$43{,}000\text{ ft} \cdot \frac{1\text{ mi}}{5280\text{ ft}} = 8.1\text{ mi}$$

There are two important comments to make here. First, since any conversion factor can be written two ways (depending on which quantity is placed in the numerator), how do we know which way to write the conversion factor? Well, we know from algebra that when we have quantities in the numerator of a fraction that are multiplied, and quantities in the denominator of the fraction that are multiplied, any quantities that appear in both the numerator and denominator cancel. Most units of measure are mathematically treated as multiplied quantities that can be cancelled out.[2] In the example above, we desire that "feet" in the given quantity (which is in the numerator) cancels out, so the conversion factor is written with feet in the denominator and miles in the numerator.

Second, if you perform the calculation above, the result that appears on your calculator screen is 8.143939394. So why didn't I write down all those digits in my result? Why did I round my answer off to simply 8.1 miles? The answer to that question has to do with the *sig-*

2 An example of a unit that cannot always be treated this way is the degree Fahrenheit.

nificant digits in the value 43,000 ft that we started with. We address the issue of significant digits later in this chapter, but in the examples that follow I always write the results with the correct number of significant digits for the values involved in the problem.

There are several important techniques you must use to help you perform unit conversions correctly; these are illustrated below with examples. You should rework each of the examples on your own paper as practice to make sure you can do them correctly. As a reminder, the conversion factors used in the examples below are all listed in Appendix A. You should study Appendix A to see which ones you must know by memory and which ones are provided to you on quizzes.

1 Use only horizontal bars in your unit fractions. Never use slant bars.

In printed materials, one often sees values written with a slant fraction bar in the units, as in the value 35 m/s. Although writing the units this way is fine for a printed document, you should not write values this way when you are performing unit conversions. This is because it is easy to get confused and not notice that one of the units is in the denominator in such an expression (s, or seconds, in my example), and the conversion factors used must take this into account.

▼ Example 2.1

Convert 57.66 mi/hr into m/s.

Writing the given quantity with a horizontal bar makes it clear that "hour" is in the denominator. This helps you to write the hour-to-seconds factor correctly.

$$57.66\ \frac{\text{mi}}{\text{hr}} \cdot \frac{1609\ \text{m}}{\text{mi}} \cdot \frac{1\ \text{hr}}{3600\ \text{s}} = 25.77\ \frac{\text{m}}{\text{s}}$$

Now that you have your result, you may write it as 25.77 m/s if you wish, but do not use slant fraction bars in the units when you are working out the unit conversion.

▲

2 The term "per" implies a fraction.

Some units of measure are commonly written with a "p" for "per," such as mph for miles per hour or gps for gallons per second. Change these expressions to fractions with horizontal bars when you work out the unit conversion.

▼ Example 2.2

Convert 472.15 gps to L/hr.

When you write down the given quantity, change the gps to gal/s, and write these units with a horizontal bar:

$$472.15\ \frac{\text{gal}}{\text{s}} \cdot \frac{3.785\ \text{L}}{1\ \text{gal}} \cdot \frac{3600\ \text{s}}{1\ \text{hr}} = 6{,}434{,}000\ \frac{\text{L}}{\text{hr}}$$

▲

3 Use the $\boxed{\times}$ and $\boxed{\div}$ keys correctly when entering values into your calculator.

When dealing with several numerator terms and several denominator terms, multiply all the numerator terms together first, hitting the $\boxed{\times}$ key between each, then hit the $\boxed{\div}$ key and enter all the denominator terms, hitting the $\boxed{\div}$ key between each. This way you do not need to write down intermediate results and you do not need to use any parentheses.

Example 2.3

Convert 43.17 mm/hr into km/yr.

The setup with all the conversion factors is as follows:

$$43.17\ \frac{\text{mm}}{\text{hr}}\cdot\frac{1\text{ m}}{1000\text{ mm}}\cdot\frac{1\text{ km}}{1000\text{ m}}\cdot\frac{24\text{ hr}}{1\text{ day}}\cdot\frac{365\text{ day}}{1\text{ yr}}=0.378\ \frac{\text{km}}{\text{yr}}$$

To execute this calculation in your calculator, enter the values and operations in this sequence:

$$43.17\times24\times365\div1000\div1000=$$

4 When converting units for area and volume such as cm^2 or m^3, use the appropriate length conversion factor twice for areas or three times for volumes.

The unit "cm^2" for an area means the same thing as "cm × cm." Likewise, "m^3" means "m × m × m." So when you use a length conversion factor such as 100 cm = 1 m or 1 in = 2.54 cm, you must use it twice to get squared units (areas) or three times to get cubed units (volumes).

Example 2.4

Convert 3,550 cm^3 to m^3.

$$3550\text{ cm}^3\cdot\frac{1\text{ m}}{100\text{ cm}}\cdot\frac{1\text{ m}}{100\text{ cm}}\cdot\frac{1\text{ m}}{100\text{ cm}}=0.00355\text{ m}^3$$

Notice in Example 2.4 that the unit cm occurs three times in the denominator, giving us cm^3 when they are all multiplied together. This cm^3 term in the denominator cancels with the cm^3 term in the numerator. And since the m unit occurs three times in the numerator, they multiply together to give us m^3 for the units in our result. Notice also that the denominator is $100\cdot100\cdot100$ = 1,000,000. This is why I write in Section 2.1.1 that to convert from cm^3 to m^3 we just divide by 1,000,000. Pay attention to this and don't make the common (and silly) mistake of dividing by 100!

This issue only arises when you have a unit raised to a power, such as when using a length unit to represent an area or a volume. When using a conversion factor such as 3.785 L = 1 gal, the units of measure are written using units that are strictly volumetric (liters and gallons), and are not obtained from lengths the way in^2, ft^2, cm^3, and m^3 are. Another common unit that uses a power is acceleration, which has units of m/s^2 in the MKS unit system.

▼ Example 2.5

Convert 5.85 mi/hr^2 into MKS units.

$$5.85\ \frac{\text{mi}}{\text{hr}^2}\cdot\frac{1609\ \text{m}}{1\ \text{mi}}\cdot\frac{1\ \text{hr}}{3600\ \text{s}}\cdot\frac{1\ \text{hr}}{3600\ \text{s}}=0.000726\ \frac{\text{m}}{\text{s}^2}$$

With this example you see that since the "hour" unit is squared in the given quantity, the conversion factor converting hours to seconds must appear twice in the conversion calculation.

▲

2.1.4 Accuracy and Precision

The terms *accuracy* and *precision* refer to the limitations inherent in making measurements. Science is all about investigating nature and to do that we must make measurements. Accuracy relates to *error*, which is the difference between a measured value and the true value. The lower the error is in a measurement, the better the accuracy. Error can be caused by a number of different factors, including human mistakes, malfunctioning equipment, incorrectly calibrated instruments, or unknown factors that influence a measurement without the knowledge of the experimenter. All measurements contain error because (alas!) perfection is simply not a thing we have access to in this world.

Precision refers to the resolution or degree of "fine-ness" in a measurement. The limit to the precision obtained in a measurement is ultimately dependent on the instrument used to make the measurement. If you want greater precision, you must use a more precise instrument. The precision of a measurement is indicated by the number of *significant digits* (or significant figures) included when the measurement is written down (see next section).

Figure 2.1 is a photograph of a machinist's rule and an architect's scale set side by side. Since the marks on the two scales line up consistently, these two scales are equally accurate. But the machinist's rule (on top) is more precise. The architect's scale is marked in 1/16-inch increments, but the machinist's rule is marked in 1/64-inch increments.

It is important that you are able to distinguish between accuracy and precision. Here is an example to illustrate the difference. Let's say Shana and Marius each buy digital thermometers for their homes. The thermometer Shana buys cost $10 and measures to the nearest 1°F. Marius pays $40 and gets one that reads to the nearest 0.1°F. Note that on a day when the actual temperature is 95.1°F, if the two thermometers are reading accurately Shana's thermometer reads 95° and Marius' reads 95.1°. Thus, Marius' thermometer is more precise.

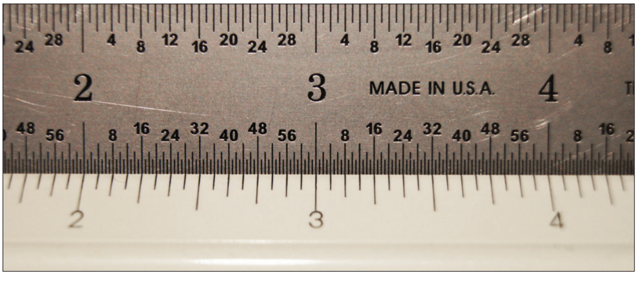


Figure 2.1. The accuracy of these two scales is the same, but the machinist's rule on the top is more precise.

Now suppose Shana reads the directions and properly installs the sensor for her new thermometer in the shade. Marius doesn't read the directions and mounts his sensor in the direct sunlight, which causes a significant error in the measurement for much of the day. The result is that Shana has lower-precision, higher-accuracy measurements!

2.1.5 Significant Digits

The precision in any measurement is indicated by the number of *significant digits* it contains. Thus, the number of digits we write in any measurement we deal with in science is very important. The number of digits is meaningful because it shows the precision present in the instrument used to make the measurement.

Let's say you are working a computational exercise in a science book. The problem tells you that a person drives a distance of 110 miles at an average speed of 55 miles per hour and wants you to calculate how long the trip takes. The correct answer to this problem *will be different* from the correct answer to a similar problem with given values of 110.0 miles and 55.0 miles per hour. And if the given values are 110.0 miles and 55.00 miles per hour, the correct answer is different yet again. Mathematically, of course, all three answers are the same. If you drive 110 miles at 55 miles per hour, the trip takes two hours. But scientifically, the correct answers to these three problems are different: 2.0 hours, 2.00 hours, and 2.000 hours, respectively. The difference between these cases is in the precision indicated by the given data, which are *measurements*. (Even though this is just a made-up problem in a book and not an actual measurement someone made in an experiment, the given data are still measurements. There is no way to talk about distances or speeds without talking about measurements, even if the measurements are only imaginary or hypothetical.)

When you perform a calculation with physical quantities (measurements), you cannot simply write down all the digits shown by your calculator. The precision inherent in the measurements used in a computation governs the precision in any result you calculate from those measurements. And since the precision in a measurement is indicated by the number of significant digits, data and calculations must be written with the correct numbers of significant digits. To do this, you need to know how to count significant digits and you must use the correct number of significant digits in all your calculations and experimental data.

Correctly counting significant digits involves four different cases:

1. Rules for determining how many significant digits there are in a given measurement.
2. Rules for writing down the correct number of significant digits in a measurement you are making and recording.
3. Rules for computations you perform with measurements—multiplication and division.
4. Rules for computations you perform with measurements—addition and subtraction.

In this course, we do not use the rules for addition and subtraction, so we leave those for a future course (probably chemistry). We now address the first three cases, in order.

Case 1 We begin with the rule for determining how many significant digits there are in a given measurement value. The rule is as follows:

> The number of significant digits (or figures) in a number is found by counting all the digits from left to right beginning with the first nonzero digit on the left. When no decimal is present, trailing zeros are not considered significant.

Let's apply this rule to several example values to see how it works:

15,679 This value has five significant digits.

21.0005 This value has six significant digits.

37,000 This value has only two significant digits because when there is no decimal trailing zeros are not significant. Notice that the word *significant* here is a reference to the *precision* of the measurement, which in this case is rounded to the nearest thousand. The zeros in this value are certainly *important*, but they are not *significant* in the context of precision.

0.0105 This value has three significant digits because we start counting with the first nonzero digit on the left.

0.001350 This value has four significant digits. Trailing zeros count when there is a decimal.

The significant digit rules enable us to tell the difference between two measurements such as 13.05 m and 13.0500 m. Mathematically, of course, these values are equivalent. But they are different in what they tell us about the process of how the measurements were made. The first measurement has four significant digits. The second measurement is more precise. It has six significant digits and would come from a more precise instrument.

Now, just in case you are bothered by the zeros at the end of 37,000 that are not significant, here is one more way to think about significant digits that may help. The precision in a measurement depends on the instrument used to make the measurement. If we express the measurement in different units, this should not change the precision. A measurement of 37,000 grams is equivalent to 37 kilograms. Whether we express this value in grams or kilograms, it still has two significant digits.

Case 2 The second case addresses the rules that apply when you record a measurement yourself, rather than reading a measurement someone else has made. When you take measurements yourself, as you do in laboratory experiments, you need to know the rules for which digits are significant in the reading you are taking on the measurement instrument. The rule for taking measurements depends on whether the instrument you are using is a digital instrument or an analog instrument. Here are the rules for these two possibilities:

Rule 1 for digital instruments

For the digital instruments commonly found in high school or undergraduate science labs, assume all the digits in the reading are significant, except leading zeros.

Rule 2 for analog instruments

The significant digits in a measurement include all the digits known with certainty, plus one digit at the end that must be estimated between the finest marks on the scale of your instrument.

The first of these rules is illustrated in Figure 2.2. The reading on the left has leading zeros, which do not count as significant. Thus, the first reading has three significant digits.

Figure 2.2. With digital instruments, all digits are significant except leading zeros. Thus, the numbers of significant digits in these readings are, from left to right, three, three, five, and five.

The second reading also has three significant digits. The third reading has five significant digits.

The fourth reading also has five significant digits because with a digital display, the only zeros that don't count are the leading zeros. Trailing zeros are significant with a digital instrument. However, when you write this measurement down, you must write it in a way that shows those zeros to be significant. The way to do this is by using scientific notation. Thus, the right-hand value in Figure 2.2 must be written as 4.2000×10^4.

Dealing with digital instruments is actually more involved than the simple rule above implies, but the issues involved go beyond what we typically deal with in introductory or intermediate science classes. So, simply take your readings and assume that all the digits in the reading except leading zeros are significant.

Now let's look at some examples illustrating the rule for analog instruments. Figure 2.3 shows a machinist's rule being used to measure the length in millimeters (mm) of a brass block. We know the first two digits of the length with certainty; the block is clearly between 31 mm and 32 mm long. We have to estimate the third significant digit. The scale on the rule is marked in increments of 0.5 mm. Comparing the edge of the block with these marks, I would estimate the next digit to be a 6, giving a measurement of 31.6 mm. Others might estimate the last digit to be 5 or 7; these small differences in the last digit are unavoidable because the last digit is estimated. Whatever you estimate the last digit to be, two digits of this measurement are known with certainty, the third digit is estimated, and the measurement has three significant digits.

The photograph in Figure 2.4 shows a measurement in milliliters (mL) being taken with a piece of apparatus called a *buret*—a long glass tube used for measuring liquid volumes. Notice in this figure that when measuring liquid volume the surface of the liquid curls up at the edge of the cylinder. This curved surface is called a *meniscus*. The liquid measurement must be made at the bottom of the meniscus for most liquids, including water. The scale on the buret shown is marked in increments of 0.1 mL. This means we estimate to the nearest 0.01 mL. To one person, the bottom of the meniscus (the black curve) may appear to be just below 2.2 mL, so that person would call this measurement 2.21 mL. To someone else, it may seem that the bottom of the meniscus is right on 2.2, in which case that person would call the reading 2.20 mL. Either way, the reading has three significant digits and the last digit is estimated to be either 1 or 0.

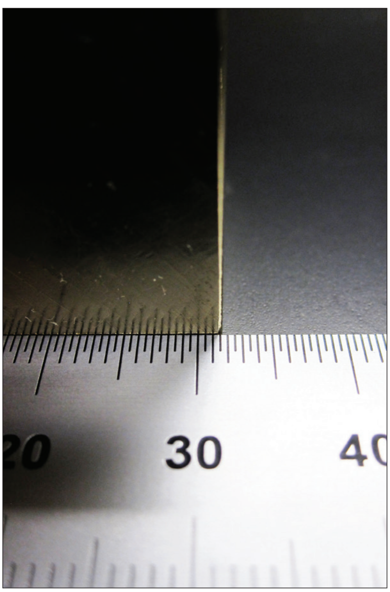


Figure 2.3. Reading the significant digits with a machinist's rule.

As a third example, Figure 2.5 shows a liquid volume measurement being taken with a piece of apparatus called a *graduated cylinder*. (We use graduated cylinders in an experiment we perform later on in this course.) The scale on the graduated cylinder shown is marked in increments of 1 mL. In the photo, the entire meniscus appears silvery in color with a black curve at the bottom. For the liquid shown in the figure, we know the first two digits of the volume measurement with certainty because the reading at the bottom of the meniscus is clearly between 82 mL and 83 mL. We have to estimate the third digit, and I would estimate the black line to be at 40% of the distance between 82 and 83, giving a reading of 82.4 mL. Someone else might read 82.5 mL, or even 82.6 mL.

It is important for you to keep the significant digits rules in mind when you are taking measurements and entering data for your lab reports. The data in your lab journal and the values you use in your calculations and report must correctly reflect the use of the significant digits rules as they apply to the actual instruments you use to take your measurements. Note also the helpful fact that when a measurement is written in scientific notation, the digits written in the stem (the numerals in front of the power of 10) *are* the significant digits.

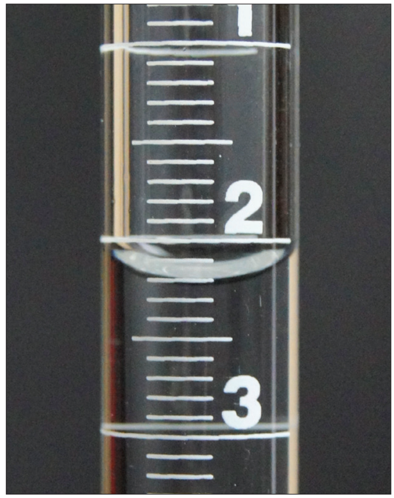


Figure 2.4. Reading the significant digits on a buret.

Case 3 The third case of rules for significant digits applies to the calculations (multiplication and division) you perform with measurements. The main idea behind the rule for multiplying and dividing is that the precision you report in your result cannot be higher than the precision you have in the measurements to start with. The precision in a measurement depends on the instrument used to make the measurement, nothing else. Multiplying and dividing things cannot improve that precision, and thus your results can be no more precise than the measurements that go into the calculations. In fact, your result can be no more precise than the *least precise value* used in the calculation. The least precise value is, so to speak, the "weak link" in the chain, and a chain is no stronger than its weakest link.

Figure 2.5. Reading the significant digits on a graduated cylinder.

There are two rules for combining the measured values into calculated values, including any unit conversions that must be performed. Here are the two rules for using significant digits in our calculations in this course:

Rule 1

Count the significant digits in each of the values you use in a calculation, including the conversion factors you use. (Exact conversion factors are not considered.) Determine how many significant digits there are in the least precise of these values. The result of your calculation must have this same number of significant digits.

Rule 1 is the rule for multiplying and dividing, which is what most of our calculations entail. (As I mentioned previously, there is another rule for adding and subtracting that you will learn when you take chemistry.)

Rule 2

When performing a multi-step calculation, you must keep at least one extra digit during intermediate calculations and round off to the final number of significant digits you need at the very end. This practice ensures that small round-off errors don't add up during the calculation. This extra digit rule also applies to unit conversions performed as part of the computation.

As I present example problems in the coming chapters, I frequently refer to these rules and show how they apply to the example at hand. As you take your quizzes, your instructor might give you a few weeks to practice and master the correct use of significant digits without penalizing you for mistakes. But get this skill down as soon as you can because soon you must use significant digits correctly in your computations to obtain the highest scores on your quizzes.

2.1.6 Scientific Notation

You have probably studied scientific notation before. However, in this course you must master it, including the use of the special key found on scientific calculators for working with values in scientific notation. Mastery of scientific notation is important because working with values in scientific notation is a basic and common occurrence in scientific work. We review the basic principles next.

Mathematical Principles Scientific notation is a way of expressing very large or very small numbers without all the zeros, unless the zeroes are *significant*. This is of enormous benefit when one is dealing with a value such as 0.0000000000001 cm (the approximate diameter of an atomic nucleus). The basic idea will be clear from a few examples.

Let's say we have the value 3,750,000. This number is the same as 3.75 million, which can be written as $3.75 \times 1{,}000{,}000$. Now, 1,000,000 itself can be written as 10^6 (which means one followed by six zeros), so our original number can be expressed equivalently as 3.75×10^6. This expression is in scientific notation. The numerals in front, the stem, are always written as one digit followed by a decimal and the other digits. The multiplied 10 raised to a power has the effect of moving the decimal over as many places as necessary to recreate our original number.

As a second example, the current population of earth is about 7,290,000,000, or 7.29 billion. One billion has nine zeros, so it can be written as 10^9. So we can express the population of earth in scientific notation as 7.29×10^9.

When dealing with extremely small numbers such as 0.000000016, the process is the same, except the power on the 10 is negative. The easiest way to think of it is to count how many places the decimal in the value must be moved over to get 1.6. To get 1.6, the decimal has to be moved to the right eight places, so we write our original value in scientific notation as 1.6×10^{-8}.

Using Scientific Notation with a Scientific Calculator All scientific calculators have a key for entering values in scientific notation. This key is labeled EE or EXP on most calculators, but others use a different label.[3] It is *very* common for those new to scientific calculators to use this key incorrectly and obtain incorrect results. So read carefully as I outline the general procedure.

The whole point of using the [EE] key is to make keying in the value as quick and error-free as possible. When using the scientific notation key to enter a value, you do not press the [×] key, nor do you enter the 10. The scientific calculator is designed to reduce all this key entry, and the potential for error, by use of the scientific notation key. You only enter the stem of the value and the power on the ten and let the calculator do the rest.

Here's how. To enter a value, simply enter the digits and decimal in the stem of the number, then hit the [EE] key, then enter the power on the ten. The value is now entered and you may do with it as you wish. As an example, to multiply the value 7.29×10^9 by 25 using a standard scientific calculator, the sequence of key strokes is as follows:

7.29 [EE] 9 [×] 25 [=]

Notice that between the stem and the power the only key pushed is the [EE] key.

When entering values in scientific notation with negative powers on the 10, the [+/−] key is used before the power to make the power negative. Thus, to divide 1.6×10^{-8} by 36.17, the sequence of key strokes is:

1.6 [EE] [+/−] 8 [÷] 36.17 [=]

Again, neither the "10" nor the "×" sign that comes before it is keyed in. The [EE] key has these built in.

Students sometimes wonder why it is incorrect to use the [10^x] key for scientific notation. To execute 7.29×10^9 times 25, they are tempted to enter the following:

7.29 [×] [10^x] 9 [×] 25 [=]

The answer is that sometimes this works, and sometimes it doesn't, and calculator users must use key entries that *always* work. The scientific notation key ([EE]) keeps a value in scientific notation all together as one number. That is, when the [EE] key is used, then to the calculator 7.29×10^9 is not two numbers, it is a single numerical value. But when the [×] key is manually inserted, the calculator treats the numbers separated by the [×] key as two separate values. This causes the calculator to render an *incorrect* answer for a calculation such as

$$\frac{3.0 \times 10^6}{1.5 \times 10^6}$$

The denominator of this expression is exactly half the numerator, so the value of this fraction is obviously 2. But when using the [10^x] key, the 1.5 and the 10^6 in the denominator are separated and treated as separate values. The calculator then performs the following calculation:

$$\frac{3.0 \times 10^6}{1.5} \times 10^6$$

3 One infuriating model uses the extremely unfortunate label [$x10^x$] which looks a *lot* like [10^x], a different key with a completely different function.

This comes out to 2,000,000,000,000 (2×10^{12}), which is not the same as 2!

The bottom line is that the EE key, however it may be labeled, is the correct key to use for scientific notation.

2.1.7 Problem Solving Methods

Organizing problems on your paper in a reliable and orderly fashion is an essential practice. Physics problems can get very complex, and proper solution practices can often make the difference between getting most or all of the points for a problem and getting few or none. Each time you start a new problem, you must set it up and follow the steps according to the outline presented in the box on pages 36 and 37, entitled *Universal Problem Solving Method*. It is very important that you always show all your work. Do not give in to the temptation to skips steps or take shortcuts. Develop correct habits for problem solving and stick with them!

2.2 Motion

In this course, we address two types of *motion*: motion at a constant *velocity*, when an object is not accelerating, and motion with a *uniform acceleration*. Defining these terms is a lot simpler if we stick to motion in one dimension, that is, motion in a straight line. So in this course, this is what we will do.

2.2.1 Velocity

Figure 2.6. A car traveling with the cruise control on is an example of an object moving with constant velocity.

When thinking about motion, one of the first things we must consider is how fast an object is moving. The common word for how fast an object is moving is *speed*. A similar term is the word *velocity*. For the purposes of this course, you may treat these two terms as synonyms. The difference is technical. Technically, the term velocity means not only *how fast* an object is moving, but also in what *direction*. The term speed refers only to how fast an object is moving. But since we are only going to consider motion in one direction at a time, we can use the terms *speed* and *velocity* interchangeably.

An important type of motion is motion at a constant velocity, like a car with the cruise control on (Figure 2.6). At a constant velocity, the velocity of an object is defined as the distance the object travels in a certain period of time. Expressed mathematically, the velocity, v, of an object is calculated as

$$v = \frac{d}{t}$$

The velocity is calculated by dividing the distance the object travels, d, by the amount of time, t, it takes to travel that distance. So, if you walk 5.0 miles in 2.0 hours, your velocity is v = (5.0 miles)/(2.0 hours), or 2.5 miles per hour.

Notice that for a given length of time, if an object covers a greater distance it is moving with a higher velocity. In other words, the velocity is proportional to the distance traveled in a certain length of time. When performing calculations using the SI System of units, distances are measured in meters and times are measured in seconds. This means the units for a velocity are meters per second, or m/s.

The relationship between velocity, distance, and time for motion at a constant velocity is shown graphically in Figure 2.7. Travel time is shown on the horizontal axis and distance traveled is shown on the vertical axis. The steeper curve[4] shows distances and times for an object moving at 2 m/s. At a time of one second, the distance traveled is two meters because the object is moving at two meters per second (2 m/s). After two seconds at this speed, the object has moved four meters: (4 m)/(2 s) = 2 m/s. And after three seconds, the object has moved six meters: (6 m)/(3 s) = 2 m/s.

The right-hand curve in Figure 2.7 represents an object traveling at the much slower velocity of 0.5 m/s. At this speed, the graph shows that an object travels two meters in four seconds, four meters in eight seconds, and so on.

To see this algebraically, look again at the velocity equation above. If we multiply both sides of this equation by the time, *t*, and cancel, we have

$$d = vt$$

This is the same equation, just written in a different form. It still applies to objects moving at a constant velocity. Written this way, t is the independent variable, d is the dependent variable, and v serves as the slope of the line relating d to t. With this form of the velocity equation, we can calculate how far an object travels in a given amount of time, assuming the object is moving at a constant velocity.

Now we work a couple of example problems, following the problem-solving method described on pages 36–37. And remember, all the unit conversion factors you need are listed in Appendix A.

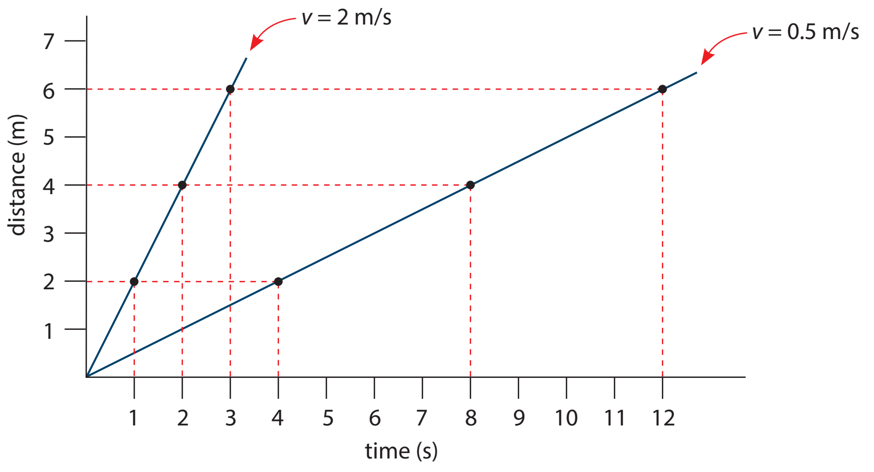


Figure 2.7. A plot of distance versus time for an object moving at constant velocity. Two different velocity cases are shown.

4 Note that when discussing graphs, the lines or curves on the graph are all referred to as *curves*, whether they are curved or straight.

Universal Problem Solving Method

Solid Steps to Reliable Problem Solving

In *Introductory Physics*, you learn how to use math to solve scientific problems. Developing a sound and reliable method for approaching problems is very important. The problem solving method shown below is used in scientific work everywhere. You must follow every step closely and show all your work.

1. Write down the given quantities at the left side of your paper. Include the variable quantities given in the problem statement and the variable you must solve for. Make a mental note of the precision in each given quantity.
2. For each given quantity that is not already in MKS units, work immediately to the right of it to convert the units of measure into MKS units. To help prevent mistakes, always use horizontal fraction bars in your units and unit conversion factors. Write the results of these unit conversions with one extra digit of precision over what you need in your final result.
3. Write the standard form of the equation you will use to solve the problem.
4. If necessary, use algebra to get the variable you are solving for alone on the left side of the equation. Never put values into the equation until this step is done.
5. Write the equation again with the values in it, using only MKS units, and compute the result.
6. If you are asked to state the answer in non-MKS units, perform the final unit conversion now.
7. Write the result with the correct number of significant digits and the correct units of measure.
8. Check your work.
9. Make sure your result is reasonable.

Example Problem

If you want a complete and happy life, do 'em just like this!

A car is traveling at 35.0 mph. The driver then accelerates uniformly at a rate of 0.15 m/s^2 for 2 minutes and 10.0 seconds. Determine the final velocity of the car in mph.

Step 1 Write down the given information in a column down the left side of your page, using horizontal lines for the fraction bars in the units of measure.

$$v_i = 35.0\ \frac{\text{mi}}{\text{hr}}$$

$$a = 0.15\ \frac{\text{m}}{\text{s}^2}$$

$$t = 2\ \text{min}\ 10.0\ \text{s}$$

$$v_f = ?$$

Step 2 Perform the needed unit conversions, writing the conversion factors to the right of the given quantities you wrote in the previous step.

$$v_i = 35.0\ \frac{\text{mi}}{\text{hr}} \cdot \frac{1609\ \text{m}}{\text{mi}} \cdot \frac{1\ \text{hr}}{3600\ \text{s}} = 15.6\ \frac{\text{m}}{\text{s}}$$

$$a = 0.15\ \frac{\text{m}}{\text{s}^2}$$

$$t = 2\ \text{min}\ 10.0\ \text{s} = 130.0\ \text{s}$$

$$v_f = ?$$

Step 3 Write the equation you will use in its standard form.

$$a = \frac{v_f - v_i}{t}$$

Step 4 Perform the algebra necessary to get the unknown you are solving for alone on the left side of the equation.

$$a = \frac{v_f - v_i}{t}$$

$$at = v_f - v_i$$

$$v_f = v_i + at$$

Step 5 Using only values in MKS units, insert the values and compute the result.

$$v_f = v_i + at = 15.6\ \frac{\text{m}}{\text{s}} + 0.15\ \frac{\text{m}}{\text{s}^2} \cdot 130.0\ \text{s} = 35.1\ \frac{\text{m}}{\text{s}}$$

Step 6 Convert to non-MKS units, if required in the problem.

$$v_f = 35.1\ \frac{\text{m}}{\text{s}} \cdot \frac{1\ \text{mi}}{1609\ \text{m}} \cdot \frac{3600\ \text{s}}{1\ \text{hr}} = 78.5\ \frac{\text{mi}}{\text{hr}}$$

Step 7 Write the result with correct significant digits and units of measure.

$$v_f = 79\ \text{mph}$$

Step 8 Check over your work, looking for errors.

Step 9 Make sure your result is reasonable. First, check to see if your result makes sense. The example above is about an accelerating car, so the final velocity we calculate should be a velocity a car can have. A result like 14,000 mph is obviously incorrect. (And remember that nothing can travel faster than the speed of light, so make sure your results are reasonable in this way as well.) Second, if possible, estimate the answer from the given information and compare your estimate to your result. In step 6 above, we see that 3600/1609 is about 2, and $2 \cdot 35.1$ is about 70. Thus our result of 79 mph makes sense.

(Optional Step 10: Revel in the satisfaction of knowing that once you get this down you can work physics problems perfectly nearly every time!)

▼ Example 2.6

Sound travels 1,120 ft/s in air. How much time does it take to hear the crack of a gun fired 1,695.5 m away?

First, write down the given information and perform the required unit conversions so that all given values are in MKS units. Check to see how many significant digits your result must have and do the unit conversions with one extra significant digit. The given speed of sound has three significant digits, so we perform our unit conversions with four digits.

$$v = 1120\ \frac{\text{ft}}{\text{s}} \cdot \frac{0.3048\ \text{m}}{\text{ft}} = 341.4\ \frac{\text{m}}{\text{s}}$$
$$d = 1695.5\ \text{m}$$
$$t = ?$$

Next, write the appropriate equation to use.

$$v = \frac{d}{t}$$

Perform any necessary algebra, insert the values in MKS units, and compute the result.

$$v = \frac{d}{t}$$

$$t = \frac{d}{v} = \frac{1695.5\ \text{m}}{341.4\ \frac{\text{m}}{\text{s}}} = 4.966\ \text{s}$$

Next, round the result so that it has the correct number of significant digits. In the velocity unit conversion and in the calculated result, I used four significant digits. The given velocity has three significant digits and the given distance has five significant digits. Thus, our result must be reported with three significant digits, but all intermediate calculations must use one extra digit. This is why I used four digits. But now we are finished and our result must be rounded to three significant digits because the least precise measurement in the problem has three significant digits. Rounding our result accordingly, we have

$$t = 4.97\ \text{s}$$

The final step is to check the result for reasonableness. The result should be roughly the same as 1500/300 or 2000/400, both of which equal 5. Thus, our result makes sense.

▲

2.2.2 Acceleration

An object's velocity is a measure of how fast it is going; it is not a measure of whether its velocity is changing. The quantity we use to measure if a velocity is changing, and if so, how fast it is changing, is the *acceleration*. If an object's velocity is changing, the object is accelerating, and the value of the acceleration is the rate at which the velocity is changing.

The equation we use to calculate uniform acceleration in terms of an initial velocity v_i and a final velocity v_f is

$$a = \frac{v_f - v_i}{t}$$

where a is the acceleration (m/s^2), t is the time spent accelerating (s), and v_i and v_f are the initial and final velocities, respectively, (m/s).

Did you notice that the MKS units for acceleration are meters per second *squared* (m/s^2)? These units often drive students crazy, and we need to pause here and discuss what this means so you can sleep peacefully tonight. I wrote just above that the acceleration is the *rate* at which the velocity is changing. The acceleration simply means that the velocity is increasing by so many meters per second, per second. Now, "per" indicates a fraction, and if a velocity is changing so many meters per second, per second, we write these units in a fraction this way and simplify the expression:

$$\frac{\frac{\text{m}}{\text{s}}}{\text{s}} = \frac{\frac{\text{m}}{\text{s}}}{\frac{\text{s}}{1}} = \frac{\text{m}}{\text{s}} \cdot \frac{1}{\text{s}} = \frac{\text{m}}{\text{s}^2}$$

Because the acceleration equation results in negative accelerations when the initial velocity is greater than the final velocity, you can see that a negative value for acceleration means the object is slowing down. In future physics courses, you may learn more sophisticated interpretations for what a negative acceleration means, but in this course you are safe associating negative accelerations with decreasing velocity. In common speech, people sometimes use the term "deceleration" when an object is slowing down, but mathematically we just say the acceleration is negative.

Before we work through some examples, let's look at a graphical depiction of uniform acceleration the same way we did with velocity. Figure 2.8 shows two different acceleration curves, representing two different acceleration values. For the curve on the right, after 1 s

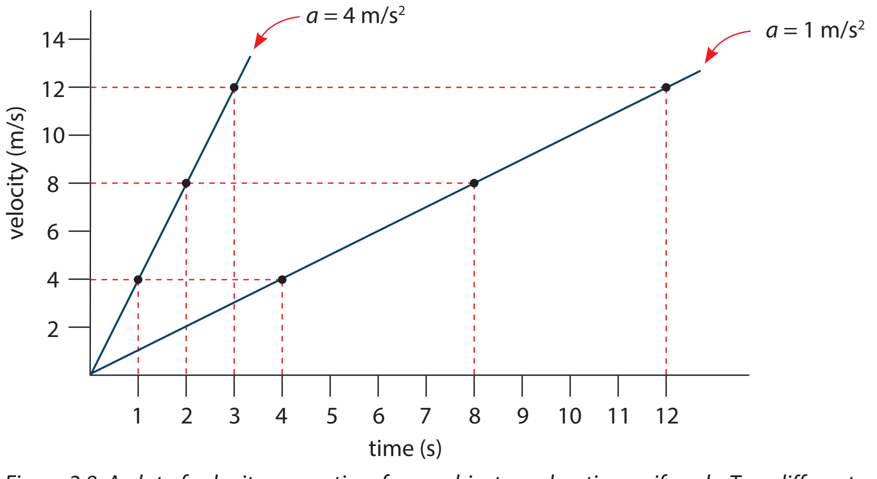


Figure 2.8. A plot of velocity versus time for an object accelerating uniformly. Two different acceleration cases are shown.

the object is going 1 m/s. After 2 s, the object is going 2 m/s. After 12 s, the object is going 12 m/s. You can take the velocity that corresponds to any length of time (by finding where their lines intersect on the curve) and calculate the acceleration by dividing the velocity by the time to get $a = 1\ \text{m/s}^2$. The other curve has a higher acceleration, $4\ \text{m/s}^2$. An acceleration of $4\ \text{m/s}^2$ means the velocity is increasing by 4 m/s every second. Accordingly, after 2 s the velocity is 8 m/s, and after 3 s, the velocity is 12 m/s. No matter what point you select on that curve, $v/t = 4\ \text{m/s}^2$.

We must be very careful to distinguish between velocity (m/s) and acceleration (m/s^2). Acceleration is a measure of how fast an object's velocity is changing. To see the difference, note that an object can be at rest ($v = 0$) and accelerating *at the same instant*.

Now, although you may not see this at first, it is important for you to think this through and understand how this counter-intuitive situation can come about. Here are two examples.

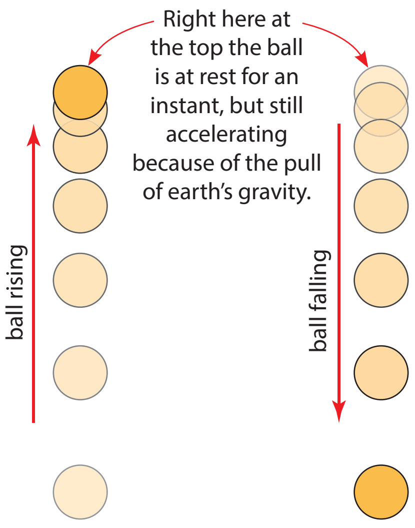


Figure 2.9. A rising and falling ball helps illustrate the difference between velocity and acceleration.

The instant an object starts from rest, such as when the driver hits the gas while sitting at a traffic light, the object is simultaneously at rest and accelerating. This is because if an object at rest is to ever begin moving, its velocity must *change* from zero to something else. In other words, the object must accelerate. Of course, this situation only holds for an instant; the velocity instantly begins changing and does not stay zero.

Perhaps my point will be easier to see with this second example. As depicted in Figure 2.9, when a ball is thrown straight up and reaches its highest point, it stops for an instant as it starts to come back down. At its highest point, the ball is simultaneously at rest and accelerating due to the force of gravity pulling it down. As before, this situation only holds for a single instant.

The point of these two examples is to help you understand the difference between the two variables we are discussing, velocity and acceleration. If an object is moving at all, then it has a velocity that is not zero. The object may or may not be accelerating. But acceleration is about whether the velocity itself is changing. If the velocity is constant, then the acceleration is zero. If the object is speeding up or slowing down, then the acceleration is not zero.

And now for another example problem, this time using the acceleration equation.

▼ Example 2.7

A truck is moving with a velocity of 42 mph (miles per hour) when the driver hits the brakes and brings the truck to a stop. The total time required to stop the truck is 8.75 s. Determine the acceleration of the truck, assuming the acceleration is uniform.

Begin by writing the givens and performing the unit conversions.

$$v_i = 42\ \frac{\text{mi}}{\text{hr}} \cdot \frac{1609\ \text{m}}{\text{mi}} \cdot \frac{1\ \text{hr}}{3600\ \text{s}} = 18.8\ \frac{\text{m}}{\text{s}}$$

$$v_f = 0$$

$$t = 8.75\ \text{s}$$

$$a = ?$$

Now write the equation and complete the problem.

$$a = \frac{v_f - v_i}{t} = \frac{0 - 18.8\ \frac{\text{m}}{\text{s}}}{8.75\ \text{s}} = -2.15\ \frac{\text{m}}{\text{s}^2}$$

The initial velocity has two significant digits, so I did the calculations with three significant digits until the end. Now we round off to two digits giving

$$a = -2.2\ \frac{\text{m}}{\text{s}^2}$$

If you keep all the digits in your calculator throughout the calculation and round to two digits at the end, you have –2.1 m/s^2. This answer is fine, too. Remember, the last digit of a measurement or computation always contains some uncertainty, so it is reasonable to expect small variations in the last significant digit. A check of our work shows the result should be about –20/10, which is –2. Thus the result makes sense.

One more point on this example: Notice that the calculated acceleration value came out negative. This was because the final velocity was lower than the initial velocity. Thus we see that a negative acceleration means the vehicle is slowing down.

▲

If you haven't yet read the example problem in the yellow Universal Problem Solving Method box, you should read it now to see a slightly more difficult example using this same equation.

2.3 Planetary Motion and the Copernican Revolution

2.3.1 Science History and the Science of Motion

People have been fascinated with the heavens since ancient times. God's people love to quote Psalm 19:

> *The heavens declare the glory of God, and the sky above proclaims his handiwork.*
> *Day to day pours out speech, and night to night reveals knowledge.*

The psalmist tells us that the glory of the stars and other heavenly bodies reveals the glory of their creator, our God. This means they convey truth to us, the truth we call General Revelation.

The study of motion has always been associated with the motion of the heavenly bodies we see in the sky, so it is particularly fitting in this chapter on motion for us to review the history of views about the solar system and the rest of the universe, referred to as "the heavens" by those in ancient times. As we will see, the particular episode known as the Copernican Revolution was a pivotal moment in that history and was the setting for the emergence of our contemporary understanding of scientific epistemology—what knowledge is and how we know what we know.

As you recall, Chapter 1 addresses the Cycle of Scientific Enterprise and examines the way science works. From that discussion you know that science is an ongoing process of modeling nature—at least that is the way we understand science now. We now understand that scientists use theories as models of the way nature works, and over time theories change and evolve as scientists learn more. Sometimes scientists find that a theory is so far off the mark that they have to toss it out completely and replace it with a different one.

The present general understanding among scientists that science is a process of modelling nature took hold around the beginning of the 20th century. The ideas that led to this understanding began to emerge at the time of the Copernican Revolution in the 16th and 17th centuries. But since natural philosophy was then entering new territory, there was a period of difficult struggle that involved both theologians and philosophers.

There are a lot of misconceptions about what happened at that time. The conflict in Galileo's day is often regarded as a fight between faith and science, and these misconceptions have led many people in today's world to the position that faith is dead and only science gives us real knowledge. But that depiction is not even close to what really happened and that belief about science is not even close to the truth. The real issue with Galileo was about epistemology. The so-called "faith versus science" debate rages today as much as ever, so it is worth spending some time to understand that crucial period in scientific history.

2.3.2 Aristotle

The study of astronomy and astrology dates back to the ancient Babylonians, but we pick up the story with the ancient Greeks and the Greek philosopher Aristotle in the 4th-century BC (Figure 2.10). Aristotle was a highly influential philosopher who wrote a lot about philosophy, physics, biology, and other fields of learning. Back then, science was called *natural philosophy* and there was really no distinction between scientists and philosophers.

That time was also many centuries before experiments became part of scientific research. Natural philosophy did involve making observations about the world, but the conclusions reached by ancient philosophers like Aristotle were based simply on observation and philosophical thought. It was still about 2,000 years before natural philosophers realized that the way things appear to our ordinary senses might not be the way they actually are and that to understand more about the world requires scientific experiments. For example, if you just walk outside and quietly look around you notice that the earth does not appear to be in motion; it feels solid and at rest. The sun, planets, and stars appear to move across the sky each day. In fact, watching a sunrise gives the distinct impression that the sun is moving up and then across the sky. Today, we understand things differently, but that is the result of the revolution we are about to explore and the experimental science that emerged at that time.

Figure 2.10. Greek philosopher Aristotle (384–322 BC).

Aristotle's ideas were grounded in the concept of *telos*—a Greek term meaning purpose, goal, or end. Aristotle believed that each thing that exists has its own *telos*, an idea we can heartily embrace today as Christians who believe that God made the world with specific purposes in mind.

Aristotle observed the serene beauty of the stars, the planets, the sun, and moon as they appear majestically to rotate around the earth day after day. He also noticed that nothing in the heavens ever seems to change. Other than the motions of the heavenly bodies, everything in the heavens seems to be pure and eternal. On earth, of course, Aristotle was surrounded by change: decay, corruption, birth, and death are all around. Animals and plants live and die, forests grow and burn, rivers flow and flood, storms come and go. These observations led Aristotle to conclude that change and corruption occur only on the earth. He wrote that imperfection and change of any kind occur only on the earth, while the heavens are pure and unchanging. Aristotle taught that the heavenly bodies—planets, stars, sun, and moon—are eternal and perfect. Further, he said that their motions must be in perfect circles since the circle is the purest and most perfect geometric shape. He conceived of the sun, moon, and planets as inhabiting celestial spheres, centered on the earth, one inside the other—an exquisite *geocentric* (earth-centered) system.

Aristotle was a tremendous moral philosopher whose ideas still have a profound influence on us today. Back in ancient times, he was regarded so highly that questioning his ideas was virtually unthinkable. Thus, his views about the heavenly motions became the basis for all further work on understanding the motions of the heavenly bodies.

2.3.3 Ptolemy

In the second century AD, the famous Alexandrian astronomer Ptolemy (Figure 2.11) worked out a detailed mathematical system based on Aristotle's ideas. (By the way, the "P" in Ptolemy is silent.) As with all ancient astronomers, Ptolemy's goal was to be able to make predictions about the movements of the planets and stars, along with other astronomical events such as eclipses, because these events were widely used as omens signifying important events on earth.

Figure 2.11. Alexandrian astronomer Ptolemy (c. AD 100–170).

Ptolemy started with Aristotle's basic ideas and developed a complex mathematical system—a model—that was quite effective in making the desired predictions. There were other astronomers around that time who developed different systems, but Ptolemy's system became the most widely accepted understanding of the heavens for over a thousand years.

2.3.4 The Ptolemaic Model

The basic structure of Ptolemy's geocentric model of the heavens is depicted in Figure 2.12. As with Aristotle, there are seven heavenly bodies, each inhabiting a *sphere* centered on the earth. Each of the heavenly bodies is also itself a perfect sphere.

The contents of the spheres are summarized in Table 2.3. The first seven spheres contain the five planets (not including the earth), the sun, and the moon. Sphere 8 contains the so-called *Firmament*, the fixed layer of stars. The stars do not move relative to each

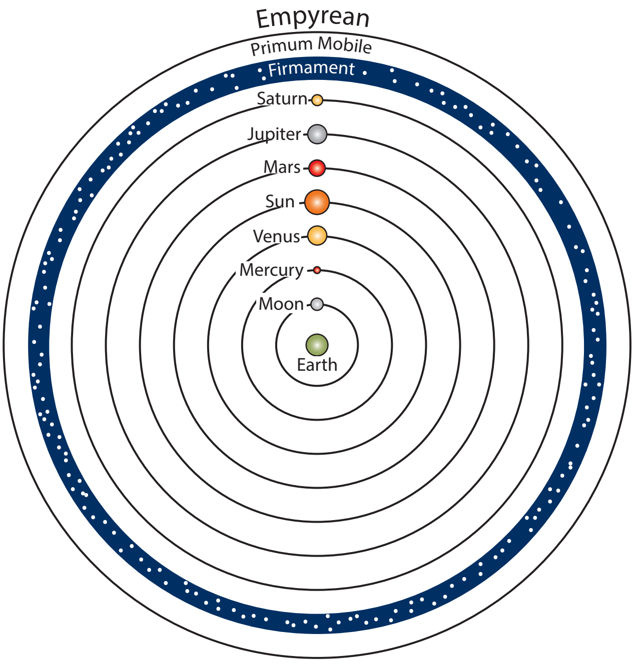


Figure 2.12. The Ptolemaic model of the heavens.

other; their positions are fixed and they rotate as a body in the eighth sphere each day. Within the firmament, the stars are arranged according to the *zodiac*, a belt of twelve constellations around the earth. The term zodiac derives from the Latin and Greek terms meaning "circle of animals," and is so named because many of the constellations in the zodiac represent animals.

The ninth sphere contains the *Primum Mobile*, which is Latin for "prime mover" (or "first mover"). The *Primum Mobile* is the sphere set into motion by God or the gods. As the *Primum Mobile* turns, it pulls all the other spheres with it, making them rotate as well. Outside the ninth sphere is the so-called *Empyrean*, the dwelling place of God or the gods.

Figure 2.12 shows the basic structure of Ptolemy's model, but there is a great deal more to the model than shown there. This is because all seven of the heavenly bodies appear to move around in the nighttime sky against the background of the fixed stars. If all the heavenly bodies simply moved in their spheres around the earth together once each day, there would be no way to account for why the planets' positions change relative to the stars.

Sphere 1	Moon
Sphere 2	Mercury
Sphere 3	Venus
Sphere 4	Sun
Sphere 5	Mars
Sphere 6	Jupiter
Sphere 7	Saturn
Sphere 8	The Firmament. This region consists of the stars arranged in their constellations according to the zodiac.
Sphere 9	The *Primum Mobile*. This Latin name means "first mover." This sphere rotates around the earth every 24 hours and drags all the other spheres with it, making them all move.
Beyond	The Empyrean. This is the region beyond the spheres. The Empyrean is the abode of God, or the gods.

Table 2.3. Contents of the spheres in the Ptolemaic model.

Ptolemy accounted for the changes by a system of *epicycles*. An epicycle is a circular planetary orbit with its center moving in a separate circular path, as depicted in Figure 2.13. As the center of an epicycle moves along its path in the sphere, the planet in the epicycle rotates about the center of the epicycle, as if the epicycle were a wheel rolling around a path centered on the earth.

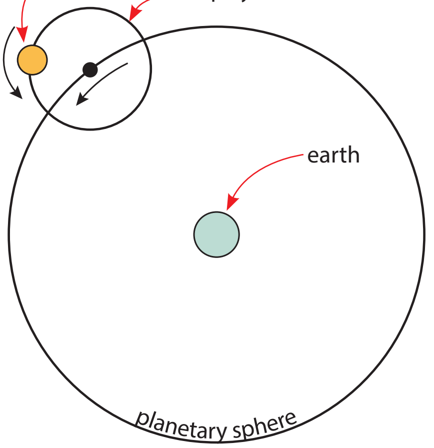


Figure 2.13. A planet moving in a path defined by an epicycle around the earth.

To help you understand why epicycles are necessary in Ptolemy's model, we discuss them in more detail in the next section. A planet moving in an epicycle moves in a path similar to a person riding in a "tea cup ride" at an amusement park, like the one picture in Figure 2.14. To account for the complex motions of the heavenly bodies, Ptolemy's model contained some 80 different epicycles. Some of the planets were located in a epicycle riding on the rim of another epicycle, which in turn moved in the sphere around the earth. Ptolemy's system was mathematically very complex, but its genius was that it worked pretty well! The main features of Ptolemy's model are summarized in the box below.

Among the different astronomers of the ancient world there were those who held to variations on this basic model. For example, some astronomers reckoned that Mercury and Venus orbited the sun while the other heavenly bodies orbited the earth. But the basic Ptolemaic model is as described in the box.

Figure 2.14. The people in the cups spin in a circle while the cup moves in a larger circle, motion like that of a planet moving on an epicycle.

The Main Principles in Ptolemy's Celestial Model

1. There are seven heavenly bodies.
2. All the heavenly bodies move in circular orbital regions called spheres. In the model, there are nine spheres plus the region beyond the spheres, with contents as listed in Table 2.3.
3. All the heavenly bodies are perfectly spherical.
4. All the spheres are centered on the earth, so this system is a *geocentric* system.
5. Corruption and change only exist on earth. All other places in the universe, including all the heavenly bodies and stars, are perfect and unchanging.
6. All the spheres containing the heavenly bodies and all the stars in the Firmament rotate completely around the earth every 24 hours.
7. Epicycles are used to explain the motion of the planets relative to the stars.

2.3.5 The Ancient Understanding of the Heavens

We soon address the new ideas that began unfolding when Nicolaus Copernicus introduced his new *heliocentric* (sun-centered) model of the heavens. But before pressing on, let's pause to consider a couple of things about the way the motion of the planets in the night sky appears to observers on earth. This will make it easier to understand why Ptolemy's system became so widely accepted.

Stationary Earth First, as I mention above, the earth does not seem to be moving. To you and I, who grew up in a time when everyone knows that the earth and other planets orbit the sun, it seems obvious that day and night are caused by the earth's rotation on its axis. We have heard about this all our lives. But stop and consider that if all we had to go on was our simple observations, it does *appear* that everything is orbiting around the earth while the earth sits still: the sun and moon rise each day, track across the sky, and set, and the planets and stars all do the same thing. Also, it doesn't feel at all like earth is rotating. We all know that anytime we spin in a circle, like people on a merry-go-round, we have to hold on to keep from falling off. We also feel the wind in our hair. Again, if we have something with us on the merry-go-round that is tall and flexible, such as a sapling, it does not remain vertical when it is moving in a circular fashion like this. Instead, it bends over because of the acceleration pulling it in its circular motion.

Figure 2.15. Greek mathematician and geographer Eratosthenes (c. 276–194 BC).

Now, the ancients knew about the large size of the earth—the Greek mathematician and geographer Eratosthenes (Figure 2.15) made a very accurate estimate of the earth's circumference—a bit under 25,000 miles—as far back as 240 BC. If a sphere that size spins in a circle once a day, the people on its surface move very fast (over 1,000 miles per hour on the equator). For this to be the case, it seemed that we would be hanging on for dear life! The trees would be laying down and we would constantly feel winds that make a hurricane seem like a calm summer day!

For all these reasons, it did not seem reasonable to believe that the apparent motion of the heavenly bodies across the sky every day was due to the earth's rotation. These arguments seemed obvious to nearly everyone before 1500, and to everyone except a few cutting-edge astronomers right up to the end of the 17th century. Only a crazy person imagined that the earth spins, and people used these arguments all the way up to the time of Galileo to prove that the earth was not orbiting the sun and spinning around once a day. Back then, these were persuasive arguments.

Forward and Retrograde Motion The second item to consider here has to do with the apparent motion of the planets in the sky against the background of the stars. If you go out and look at, say, Mars each night and make a note of its location against the stars, you see that it is in a slightly different place each night. The planet gradually works its way along in a pathway against the starry background night after night. If you track the planet for several months or a year, it moves quite far. As mentioned above, Ptolemy used epicycles to account for this *forward motion* of planets against the background of fixed stars.

Going back to watching Mars, if you follow the planet's progress long enough, you see that there are periods of time lasting several weeks when the nightly progress of the planet reverses course. Mars appears to be backing up! This apparent backing up is called *retrograde motion*. Ptolemy used epicycles to account for this, too.

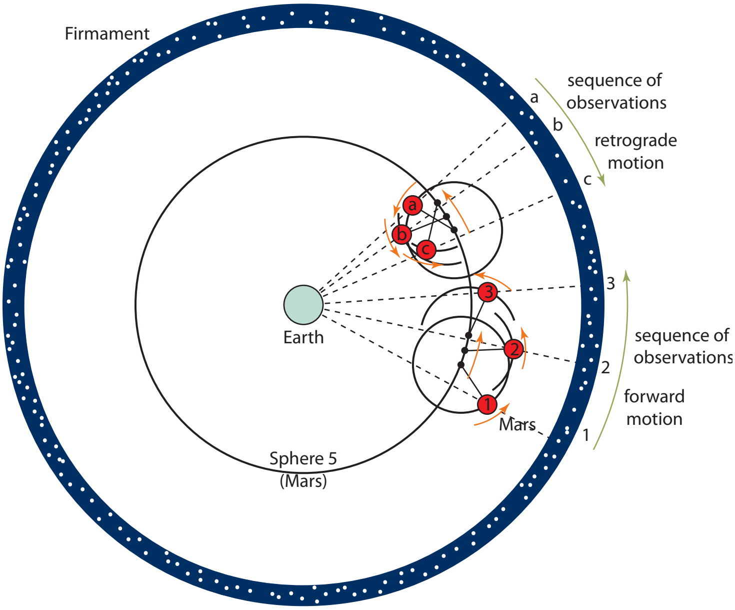


Figure 2.16. Using epicycles to explain the forward and retrograde motion of heavenly bodies against the background of fixed stars.

Figure 2.16 is a diagram showing how epicycles are used in the geocentric system to account for the planetary motions—both forward and retrograde—against the background of the fixed stars. Mars is shown in red moving on an epicycle, while the center of the epicycle moves around the earth. The dashed lines are the lines of sight from earth to Mars, and the letters and numbers outside the firmament show the locations where Mars appears among the stars at different times.

The lower right part of the diagram shows Mars in three locations (labeled 1, 2, and 3) over the course of a few weeks. Compared to the back-

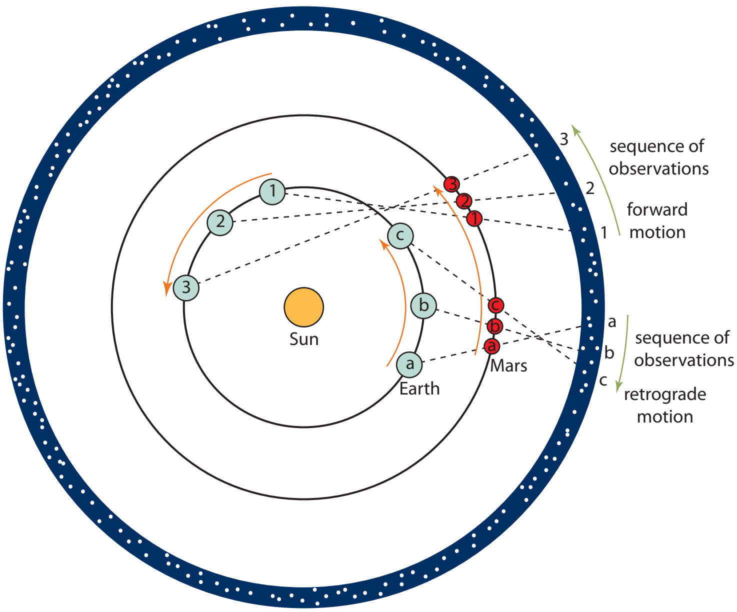


Figure 2.17. Explanation of forward and retrograde motion in the Copernican system.

ground of fixed stars, Mars exhibits forward motion during a sequence of nighttime observations.

The upper right part of the figure shows Mars' locations during a different sequence of observations (a, b, and c) some months later. Mars is now on the other side of its epicycle. The center of the epicycle continues to move in the same direction in its sphere around earth. But since Mars is on the near side of its epicycle, the sequence of observations of its location against the starry background—a, b, and c—maps along the starry background in the opposite direction. This apparent motion of Mars in the opposite direction is retrograde motion.

While we are on the subject, we may as well look at how forward and retrograde motion are explained in a *heliocentric* system—a system in which the planets orbit the sun. The system introduced by Copernicus is a heliocentric system. Assuming that the earth moves faster in its orbit than Mars (which is correct), the explanation is straightforward. As shown in the upper part of Figure 2.17, when the earth and Mars are on opposite side of their orbits, the observations of Mars' location against the stars exhibit forward motion. But when the earth and Mars are on the same side of the sun, as in the center-right part of the figure, the earth's greater velocity makes Mars' position against the stars exhibit retrograde motion.

To summarize, none of the planets actually reverses course in its orbit, and neither the geocentric nor heliocentric models depict planets as reversing direction. But depending on the system, the presence of epicycles and the relative locations of earth and a planet can combine to produce the appearance of forward or retrograde motion of the planet against the fixed background of the stars.

2.3.6 The Ptolemaic Model and Theology

We soon continue our history of the science of planetary motion by reviewing the momentous events of the 16th and 17th centuries. Between Ptolemy and Copernicus were 1,300 years of theology and philosophy. During this long period of history, a strong tradition emerged among many theologians that the Ptolemaic model of the heavens aligned very well with certain passages in the Bible. This circumstance led theologians in this tradition to assume that such passages were to be interpreted as literal descriptions of the motions of the heavenly bodies. Here are a few examples of passages that seem to describe the earth as motionless, with the sun and stars going around the earth:

> *He set the earth on its foundations, so that it should never be moved (Psalm 104:5).*
>
> *He made the moon to mark the seasons; the sun knows its time for setting (Psalm 104:19).*
>
> *[The sun's] rising is from the end of the heavens, and its circuit to the end of them (Psalm 19:6).*
>
> *The sun rises and the sun goes down, and hastens to the place where it rises (Ecclesiastes 1:5).*

Additionally, other features in the Ptolemaic model (derived from Aristotle) seemed to line up with biblical symbolism. For example:

- Seven is the biblical number symbolizing perfection, so it made sense that God's creation contains seven heavenly bodies.
- Circles are the most perfect shape, regarded as divine from the times of the ancient Greeks, so the spherical bodies inhabiting spheres in which they move seemed to reflect the perfection of their Creator.

- Corruption was thought to exist only on earth, and it seemed this was obviously because of the curse that resulted from the Fall of man.

The result of such teaching was that many theologians assumed that the biblical passages and doctrines described above, along with the Ptolemaic model of the heavens, were literal descriptions of the true nature of reality. To these theologians, anyone who had different ideas about the heavens—such as, for example, the idea that the earth moved and orbited the sun—should be censored and prevented from spreading teachings they felt were unbiblical.

Although widespread, this tradition of associating the Ptolemaic model with the Bible was by no means universal. Many theologians took a completely different position, including the great theologian and philosopher Augustine, a bishop in northern Africa in the 4th and 5th centuries AD. An insightful and relevant passage from Augustine is found in his book *On the Literal Meaning of Genesis*:

> Usually, even a non-Christian knows something about the earth, the heavens, and the other elements of this world, about the motion and orbit of the stars and even their sizes and relative positions, about the predictable eclipses of the sun and moon, the cycles of the years and the seasons, about the kinds of animals, shrubs, stones, and so forth, and this knowledge he holds to as being certain from reason and experience. Now it is a disgraceful and dangerous thing for an infidel to hear a Christian, presumably giving the meaning of Holy Scripture, talking nonsense on these topics, and we should take all means to prevent such an embarrassing situation, in which people show up vast ignorance in a Christian and laugh it to scorn. The shame is not so much that an ignorant individual is derided, but that people outside the household of faith think our sacred writers held such opinions, and, to the great loss of those for whose salvation we toil, the writers of our Scripture are criticized and rejected as unlearned men.

As we open the curtain now on the rest of our story, it is key to remember that many church theologians were strong supporters of those engaged in natural philosophy. The Roman Catholic Church—which figures prominently in these events—had a long tradition of supporting intellectual inquiry, including natural philosophy, and many of the individual theologians in the church were admirers of the scientists involved in these events.

2.3.7 Copernicus and Tycho

Nicolaus Copernicus (Figure 2.18), a Polish astronomer, first proposed a detailed, mathematical, heliocentric model of the heavens, with the earth rotating on its axis, all the planets moving in circular orbits around the sun, and the moon orbiting the earth.

Copernicus' system was about as accurate—and about as complex—as the Ptolemaic system. Copernicus' model still used circular orbits and because of this he still had to use epicycles to make the model accurate. Still, the model is an arrangement that is a lot closer to today's understanding than the Ptolemaic model is.

As mentioned in the accompanying box, Copernicus dedicated his famous work *On the Revolutions of the Heavenly Spheres* to Pope Paul III. This dedication indicates that the Roman Catholic Church itself was not opposed to Copernicus' ideas. Nevertheless, Copernicus knew there were scholars in the Church who were strongly opposed to the suggestion that the earth moved. Being a sensitive and godly man, he didn't want to cause trouble so he published his work privately to his close friends in 1514. Just before Copernicus' death in

Figure 2.18. Polish astronomer Nicolaus Copernicus (1473–1543).

1543, his student and admirer, mathematician and astronomer Georg Joachim Rheticus, persuaded Copernicus to publish the work. Rheticus delivered the manuscript to the printer and brought proofs back to Copernicus to review. Rheticus was not continuously present with the printer, and during his absence a theologian named Andreas Osiander added an unsigned "note to the reader" to the front of Copernicus' book stating that the heliocentric ideas were *hypotheses* (although *theory* is the better term, since we are talking about a *model*) that were useful for the purpose of performing computations and not descriptions of actual reality. Because of this note, people generally thought that it expressed Copernicus' own viewpoint. However, Rheticus was outraged by the addition and marked it out with a red crayon in the copies he sent to people. Copernicus did not live to see the final printed version of his book, but Rheticus' reaction to Osiander's note suggests that Copernicus regarded his model as more than merely an imaginary convenience that made computations easier.

Nicolaus Copernicus gave us a beautiful description of our Creator, one that is often quoted. In the preface to his book *On the Revolutions of Heavenly Spheres* he dedicated the book to Pope Paul III. Copernicus wrote:

"I can reckon easily enough, Most Holy Father, that as soon as certain people learn that in these books of mine which I have written about the revolutions of the spheres of the world I attribute certain motions to the terrestrial globe, they will immediately shout to have me and my opinions hooted off the stage."

Copernicus went on to review the shortcomings of the work of other astronomers, and then justified his own work:

"Accordingly, when I had meditated upon this lack of certitude in the traditional mathematics concerning the composition of movements of the spheres of the world, I began to be annoyed that the philosophers, who in other respects had made a very careful scrutiny of the least details of the world, had discovered no sure scheme for the movements of the machinery of the world, which has been built for us by the Best and Most Orderly Workman of all."

—from Nicolaus Copernicus, *On the Revolutions of Heavenly Spheres* (1543)

Tycho Brahe (Figure 2.19), was a Danish nobleman and astronomer. Tycho[5] built a magnificent observatory called the Uraniborg on an island Denmark ruled at the time. This observatory is depicted in Figure 2.20.

Figure 2.19. Danish astronomer Tycho Brahe (1546–1601).

Tycho was a passionate and hotheaded guy, as evidenced by the fact he had the bridge of his nose cut off in a duel. (You can see his prosthesis in Figure 2.19 if you look closely.) Even though Tycho's Uraniborg must have been the most palatial observatory in the world, he had a falling out with the new King of Denmark and decided to leave. In 1597, Tycho moved to Prague in Bohemia (the modern-day Czech Republic) and became Imperial Mathematician for Rudolph II, King of Bohemia and Holy Roman Emperor there. Tycho spent his life cataloging astronomical data for over 1,000 stars (with cleverly contrived instruments, but only a primitive telescope). His work was published much later (1627) by Johannes Kepler in a new star catalog that identified the positions of these stars with unprecedented accuracy.

Tycho witnessed and recorded two astronomical events that became historically very important. First, in 1563 he observed a *conjunction* between Jupiter and Saturn. A conjunction, illustrated in Figure 2.21, occurs when two planets are in a straight line with the earth so that from earth they appear to be in the same place in the sky. Tycho predicted the date for this conjunction using Copernicus' new heliocentric model. The prediction was close (this is good) but was still off by a few days (not so good). The error indicated that there was still something lacking in Copernicus' model. (There was: the orbits are not circular as Copernicus assumed.) Second, in 1572 Tycho observed what he called a "nova" (which is Latin for *new*; today we would call it a supernova) and proved that it was a new star. This discovery rocked the Renaissance world because it was strong evidence that the stars are not perfect and unchanging as Aristotle had thought and as the Ptolemaic model of the heavens declared.

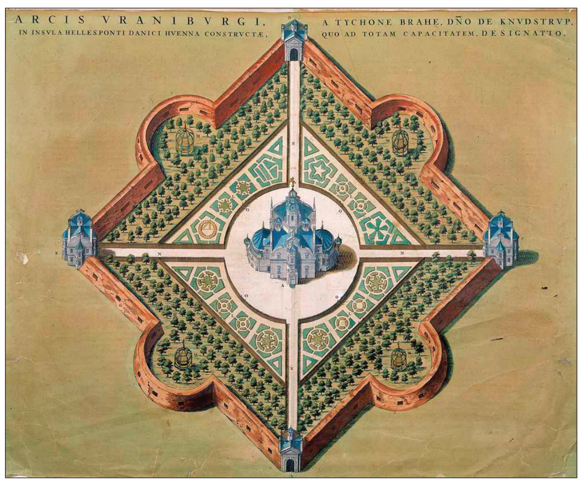


Figure 2.20. Tycho's Danish observatory, the Uraniborg.

Although familiar with Copernicus' model, Tycho was a proud advocate of his own model, in which the sun and moon orbit the earth and the other planets orbit the sun, which in turn orbits the earth. His model did have the advantage of maintaining a stationary earth, which allowed Tycho to avoid controversy with those who

5 I know it is appropriate to refer to historical figures by their last names, but most references in the literature refer to Tycho; historians rarely call him Brahe. I love the name Tycho, so I also call him that.

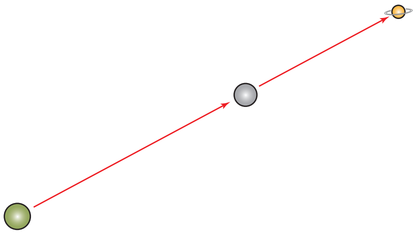


Figure 2.21. The alignment of three planets, called a conjunction.

insisted that the Bible taught that the earth did not move.

Tycho also had a good technical reason for rejecting Copernicus' model. If the earth moves in an orbit, then earth's location is different in the summer from its location in the winter. This means the relative positions of the stars should be slightly different at these different times of the year, an effect called *stellar parallax*. (As an analogy, imagine yourself looking at the trees in a forest. If you take a few steps to one side, the positions of the trees relative to each other in your new location are different.) At that time, no stellar parallax had been observed, and Tycho knew that this meant that either the earth was stationary or the stars were incredibly far away. Copernicus had accepted the great distance of the stars but Tycho did not, and famously wondered, "What purpose would all that emptiness serve?" In fact, stellar parallax was not observed until 1838, when telescopes were finally up to the task. The discovery of stellar parallax in 1838 was the *first actual evidence* that Copernicus was right. It helps to keep this in mind when we get to the controversy surrounding Galileo.

2.3.8 Kepler and the Laws of Planetary Motion

Johannes Kepler (Figure 2.22), a German astronomer and mathematician, was invited in 1600 to join the research staff at Tycho's observatory in Prague and became the Imperial Mathematician there the following year, after Tycho's death. Kepler had access to Tycho's massive body of research data and used it to develop his famous three *laws of planetary motion*, the first two of which were published in 1609. He discovered the third law a few years later and published it in 1619. Today, Kepler's laws of planetary motion remain the currently accepted model describing our solar system.

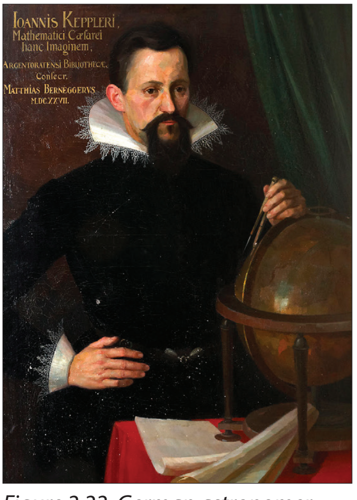


Figure 2.22. German astronomer and mathematician Johannes Kepler (1571–1630).

Kepler was a godly man and took his faith very seriously, even though he was caught in the middle during the Counter-Reformation, a time of serious disagreement between Roman Catholics and Protestants. Kepler was also an amazing scientist who believed that he had been called to glorify God through his discoveries. In addition to his astronomical discoveries, he made important discoveries in geometry and optics, he figured out some of the major principles of gravity later synthesized by Isaac Newton, and he was the first to hypothesize that the sun exerted a force on the earth.

I want to show you the three beautiful laws of planetary motion Kepler discovered. For your memory work, you may focus on remembering only the first one. But I want you to see some things about the way the solar system is designed, and the One who designed it, so I am going to describe all three of Kepler's Laws.

Kepler's first law of planetary motion is as follows:

First Law Each of the planetary orbits is an ellipse, with the sun at one focus.

A planet in an elliptical orbit is depicted in Figure 2.23. You may not have studied *ellipses* yet in math, so I will describe them. An ellipse is a geometric figure shaped like this: . An ellipse is similar to a circle, except that instead of having a single point locating the center, an ellipse has two points on either side of the center called *foci* that define its shape. (The term *foci* is plural, and pronounced FOH-sigh; the singular is *focus*.) Out in space, each planet travels on a path defined by a geometrical ellipse. The planetary orbits all have one focus located at the same place in space and this is where the sun is. Think how incredible it is that Kepler figured this out! He was a monster mathematician (no calculator!) and an extremely careful scientist, and the fact that scientists had understood the orbits to be circular for two thousand years did not get in his way. To me, this is simply amazing.

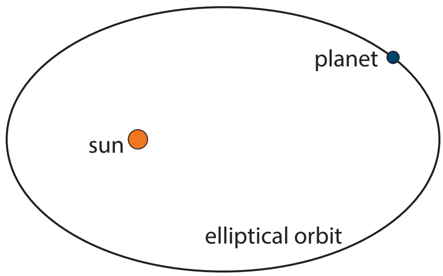


Figure 2.23. A planet in an elliptical orbit around the sun (Kepler's First Law).

Kepler's second law is not hard to understand. It is in the next box.

Second Law A line drawn from the sun to any planet sweeps out a region in space that has equal area for any equivalent length of time.

The second law is depicted in Figure 2.24. The idea is that for a given period of time, say, a month or a week or whatever, the shaded region in the figure has the same area, regardless of where the planet is in its orbit. Now, since the sun is off-center, this law implies that the planets travel faster when they are closer to the sun and slower when they are farther away. Keep thinking about how stunning it is that a guy without a calculator or any modern computer could figure this out, all from the observational data that Tycho had assembled.

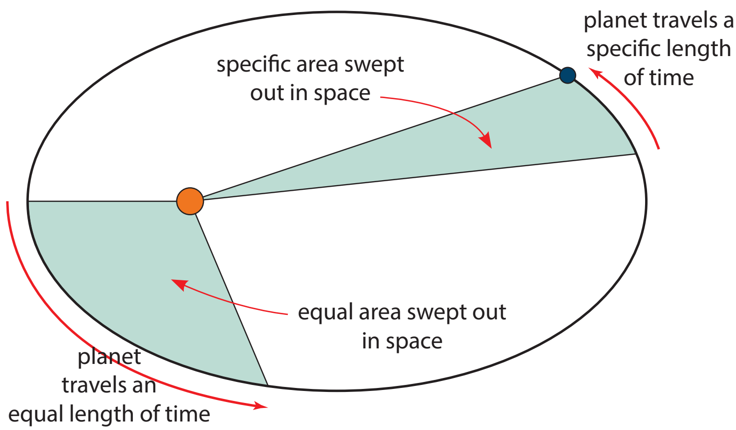


Figure 2.24. Equal areas are swept in space for equal periods of time (Kepler's Second Law).

Kepler's third law is definitely more mathematically complex than the first two. This law is shown in the next box.

Third Law The orbits of any two planets are related as follows:

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

where T_1 and T_2 are the planets' orbital periods, and R_1 and R_2 are their mean distances from the sun.

You might be relieved to learn that we will not be doing any computations with this law. However, Kepler's third law of planetary motion is a stunning example of the mathematical modeling that physicists do all the time, so I want to comment on it here for a bit.

The third law is quite accurate. The equation can be expressed in a way that shows that the orbital period, T, for any planet depends simply on the planet's mean distance from the sun, R. This expression of the third law is written as

$$T = kR^{3/2}$$

In this equation, k is simply a constant that depends on the units used for T and R. I am not planning to go crazy with the math here, and I know you may be freaking out wondering what it means to raise a variable like R to the 3/2 power. Right now it doesn't matter. You will learn all that when you get to Algebra 2. I just want to show how simple Kepler's

Johannes Kepler viewed his discoveries of the mathematical order of nature as amazing revelations given to him by God. Some of the things Kepler worked on were very strange, such as his attempt to develop a theory of the spheres associated with the five regular Platonic solids and the mathematics of musical ratios developed by the Greeks. Although those ideas were abandoned, Kepler had the courage to look carefully at the astronomical data and this led him to his discovery of the laws of planetary motion.

Read the prayer Kepler wrote at the end of his book *Harmonies of the World*:

O Thou Who dost by the light of nature promote in us the desire for the light of grace, that by its means Thou mayest transport us into the light of glory, I give thanks to Thee, O Lord Creator, Who hast delighted me with Thy makings and in the works of Thy hands have I exulted. Behold! now, I have completed the work of my profession, having employed as much power of mind as Thou didst give to me; to the men who are going to read those demonstrations I have made manifest the glory of Thy works, as much of its infinity as the narrows of my intellect could apprehend. My mind has been given over to philosophizing most correctly: if there is anything unworthy of Thy designs brought forth by me—a worm born and nourished in a wallowing place of sins—breathe into me also that which Thou dost wish men to know, that I may make the correction: If I have been allured into rashness by the wonderful beauty of Thy works, or if I have loved my own glory among men, while I am advancing in the work destined for Thy glory, be gentle and merciful and pardon me; and finally design graciously to effect that these demonstrations give way to Thy glory and the salvation of souls and nowhere be an obstacle to that.

—from Johannes Kepler, *Harmonies of the World* (1619) third law really is. Think about it. This simple equation accurately relates the period of any planet's orbit to that planet's mean distance from the sun.

Now I don't know about you, but when I see an equation that is as amazing and as simple as this, it sets me thinking. First, Kepler's work as a scientist is first class. He figured this out from data collected in the era before calculators and before computers. This was only three years after Shakespeare died!

Second, this equation says something deep about the universe we live in. The universe can be modeled with simple mathematics that can be understood by high school kids! How do you think this could be possible? Is it possible that a randomly evolving universe that occurred by chance, with no plan, could exhibit this kind of deep mathematical structure? I do not believe it is and I am not alone. Many great scientists—even non-Christian scientists—have called attention to the beautiful mathematical structure that appears everywhere in nature and have called it either a great mystery or evidence of God's handiwork. The fact that our solar system has the kind of beautiful and simple mathematical structure represented by Kepler's third law is strong evidence for an intelligent creator. This is not to say that Kepler's third law is itself the truth about nature. It is quite accurate, but as we will see below, claiming that it is the *truth* is an overstatement. But the fact that nature can be accurately modeled with mathematics by humans—even if we don't know the exact truth of nature itself—is because nature exhibits an order and regularity that can only be explained by the hand of "the Best and Most Orderly Workman of all."

2.3.9 Galileo

Galileo Galilei (Figure 2.25) worked at the university at Padua, Italy, and later as chief mathematician and philosopher for the ruling Medici family in Florence, Italy. Galileo's work in astronomy represents the climax of the Copernican Revolution. He made significant improvements to the telescope and used the telescope to see the craters on the moon and sunspots, which provided additional evidence that the heavens were not perfect and unchanging as Aristotle and Ptolemy had maintained. In 1610, he used the telescope to discover four of the moons around Jupiter, which was clearly in conflict with the idea that there had to be exactly seven heavenly bodies. He was fully on board with all the new science of the Copernican model, but, oddly, he never did accept Kepler's discovery that the planets' orbits were elliptical rather than circular. Galileo published his early astronomical discoveries in 1610 in a book called *The Starry Messenger*.

Most people know that Galileo was put on trial in 1633 by the Holy Office of the Inquisition established by Roman Catholic Church. However, the reasons for that trial are widely and seriously misunderstood. The real story is rather illuminating, and I will explain it here as briefly as possible.

Galileo is famous for this remark: "Philosophy is written in this grand book—I mean the universe—which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language in which it is written. It is written in the language of mathematics." This beautiful state-

Figure 2.25. Florentine scientist Galileo Galilei (1564–1642).

ment calls attention to the mathematical structure of creation, which, as we saw above, is strong evidence for a wise creator behind the existence of the universe. However, Galileo erred in taking his statement too far, claiming that the mathematics he used in his astronomical discoveries was the *truth* about nature. Galileo felt that his work *proved* beyond question that Copernicus was right. Galileo, along with other scientists over the next three centuries, still had to learn the limits associated with human modeling of nature.

In contrast to Galileo's attitude toward his work is the attitude of his friend Cardinal Roberto Bellarmine, an important Church official. Bellarmine was a great admirer of Galileo's work, but Bellarmine's thinking was very much along the lines of our discussion in the previous chapter: scientific inquiry leads to theories which are *models* and cannot be regarded as truths; models are provisional and subject to change. In this attitude of an important Church official, we recognize a remarkable early statement of the attitude toward scientific knowledge that today is held as the correct way to think about scientific theories. Bellarmine cautioned Galileo that no natural science could make claims as to truth and urged him to present his ideas as everyone thought Copernicus had done—as *hypotheses* rather than as *truths*.

There were definitely mistakes on both sides of the conflict that led eventually to Galileo's trial by the Holy Office. On Galileo's part, the mistake was in pushing his ideas too forcefully with the claim that they were true. Galileo's position implied that the theologians who claimed that the Bible lined up with the Ptolemaic system were wrong in their interpretation of the Bible. Galileo wrote an important letter at that time explaining that the theologians needed to reconsider their interpretations of Scripture in light of what the scientific evidence was showing. Galileo was correct in his views about interpreting Scripture, but his claims pertaining to the truth of his discoveries went too far.

To the theologians and church officials who held to the Ptolemaic view, having their views called into question was equivalent to calling the Bible itself into question. This was their mistake: they did not yet understand that the Bible has to be interpreted just as observations of nature have to be interpreted, and they were not yet ready to reconsider their views about the heavens and their interpretations of Scripture. It is interesting to note that in 1992, Pope John Paul II gave an address in which he commended Galileo's comments on the necessity of interpreting Scripture!

Recall that at this time, there was as yet no physical evidence that the earth was moving and rotating on an axis. As I mention in Section 2.3.7, the first actual evidence for earth's motion around the sun came in 1838 with the discovery of stellar parallax. Evidence for the rotation of the earth came a bit later in 1851 with the invention of Foucault's pendulum, like the one shown in Figure 2.26. The rotation of the earth causes a small change of direction in each swing of a very long, massive pendulum. If the earth were not rotating, the pendulum would swing steadily back and forth in the same direction.

Figure 2.26. A Foucault Pendulum in the Panthéon in Paris.

Now, briefly, here is the sequence of events that led to Galileo's trial. Rumors got around that Galileo had been secretly examined by the Holy Office and forced to abjure (renounce) his views about Copernicus. To help Galileo fight these annoying rumors, Cardinal Bellarmine wrote Galileo a letter in 1616 stating that the rumors were false. The letter went on to say that though Galileo had not been taken before the Holy Office, he had been told not to defend or teach as true the system of Copernicus. Then in 1632, Galileo published another major work on astronomy in which he did in fact uphold the system of Copernicus against the system of Ptolemy. The Pope at the time, Urban VIII, was also a friend and admirer of Galileo, but when he heard that Galileo had published such a book after having specifically been told not to, he was extremely upset and had no choice but to have Galileo examined by the Holy Office. This was Galileo's famous 1633 trial.

The details leading to Galileo's trial are very complex, but the controversy boils down to the two issues I have emphasized. First, Galileo pushed his scientific claims too far, claiming truth for a scientific theory which could not be regarded as more than a model of nature. Second, he published a book in defiance of an injunction against doing so. Galileo was a pious and godly man. There is good evidence that he never did actually intend to fall afoul of the injunction. But when the Holy Office persuaded him that he had, he was immediately ready to confess his actions and abjure them. This he did. Galileo was never tortured, but it was necessary that he be punished in some way. His friend Pope Urban VIII made it as easy on Galileo as he could by confining him to "house arrest" and prohibited him from further publishing. He lived for a few months in Rome in the palace of one of the cardinals, and then was allowed to return to his home in Florence were he lived in house arrest for the last eight years of his life.

In addition to his work in astronomy, Galileo developed ground-breaking ideas in physics over the course of 30 years of work. These ideas were published after his trial in what would be his final book.[6] Before Galileo, scientists had always accepted Aristotle's physics, which held that a force was needed to keep an object moving. Galileo broke with this 2,000-year-old idea and hypothesized that force was needed to *change* motion but not to *sustain* motion as Aristotle had taught. Galileo was the first to formulate the idea of a friction force that caused objects to slow down. By conducting his own experiments, Galileo also discovered that all falling objects accelerate at the same rate (the acceleration of gravity, 9.80 m/s^2), which is mathematically very close to Isaac Newton's second law of motion (our topic in the next chapter). Galileo's studies in physics thrust forward the Scientific Revolution and set the stage for the work of Isaac Newton, where the Scientific Revolution reached its climax.

The saga of the Copernican Revolution ends more or less with Galileo. Within 50 years of Galileo's death, the heliocentric model of the planetary orbits was becoming widely accepted. But while we are studying the planets and gravity, the whole story just isn't complete unless we mention two more key figures in the history of science.

2.3.10 Newton, Einstein, and Gravitational Theory

Sir Isaac Newton (Figure 2.27) is perhaps the most celebrated mathematician and scientist of all time. He was English, as his title implies, and he was truly phenomenal. He held a famous professorship in mathematics at Cambridge University. He developed calculus. He developed the famous laws of motion, which we will examine. He developed an entire theory of optics and light. He formulated the first quantitative law of gravity called the *law*

6 Since he was forbidden to publish through the Catholic Church, the book was published by a Protestant publisher in Holland.

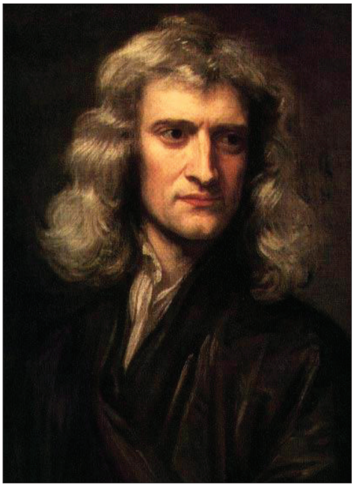


Figure 2.27. English scientist Isaac Newton (1643–1727).

of universal gravitation. His massive work on motion, gravity, and the planets, *Principia Mathematica*, was published in 1687. This work is one of the most important publications in the history of science.

In this course, we do not perform computations with Newton's law of universal gravitation, and you do not need to memorize the equation for it. But let's look at it here briefly. The law is usually written as

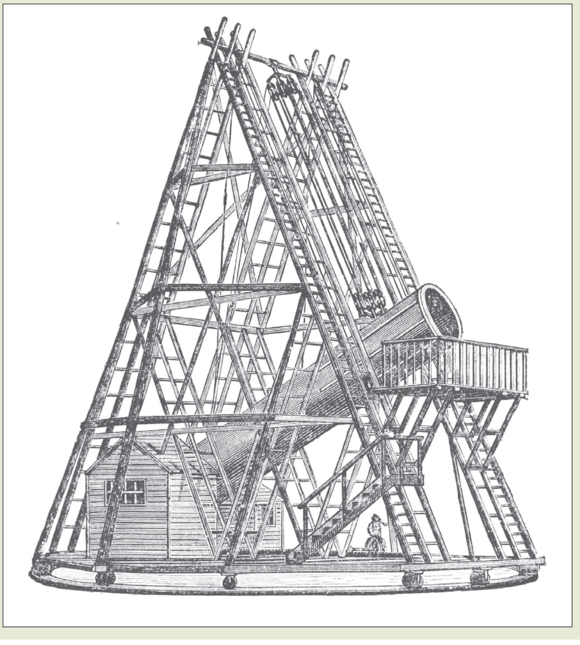
$$F = G\frac{m_1 m_2}{d^2}$$

where G is a constant, m_1 and m_2 are the masses of any two objects (such as the sun and a planet), and d is the distance between the centers of the two objects.

Newton theorized that every object in the universe pulls on every other object in the universe, which is why his law is called the law of *universal* gravitation. We now understand that he was correct. *Everything* in the universe pulls on *everything* else. I have no idea how Newton figured this out. The equation above gives the force of gravitational attraction between any two objects in the universe. Amazingly, this equation is quite accurate, too! Notice from the equation that Newton's model depends

Do You Know ... Who built the first monster telescope?

William Herschel was a German astronomer who moved to England when he was a young man. He was a major contributor to pushing the technology of the reflecting telescope to new limits, and spent vast amounts of time casting and polishing his own mirrors. He constructed the largest telescopes ever built at the time.



In 1781, Herschel discovered the planet Uranus. Herschel's sister Caroline was an important astronomer herself. She worked closely with her brother. Herschel gave her a telescope of her own and with it she discovered many new comets, for which she became recognized.

Herschel's monster 40-foot telescope, shown to the left, had a primary mirror over four feet in diameter. In 1789, on the first night of using the new telescope, Herschel discovered a new moon of Saturn. He discovered another new moon about a month later.

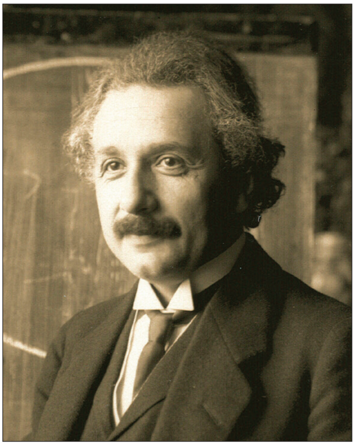


Figure 2.28. German physicist Albert Einstein (1879–1955).

on each object having mass because the force of gravity has both masses in it multiplied together. Newton's model implies that if either mass is zero, the force of gravitational attraction is zero.

While we are here looking at Isaac Newton, we should pause and consider the relationship between his physical theories (including law of universal gravitation and his laws of motion) and Kepler's mathematical theory of planetary motion. It turns out that Kepler's discovery about the elliptical orbits and the relationship between the period and mean radius of the orbit can be directly derived from Newton's theories, and Newton does derive them in *Principia Mathematica*. But Newton's equations apply much more generally than Kepler's do. As we see in the next chapter, Newton's laws apply to all objects in motion—planets, baseballs, rockets—while Kepler's laws apply to the special case of the planets' orbits. If we consider this in light of my comments in Chapter 1 about the way theories work, we see that Newton's laws explain everything Kepler's laws explain, and more. This places Newton's theory about motion and gravity above Kepler's, so Newton's theories took over as the most widely-accepted theoretical model explaining gravity and motion in general. However, even though Newton's laws ruled the scientific world for nearly 230 years, they do not tell the whole story.

This is where the German physicist Albert Einstein (Figure 2.28) comes in with his *general theory of relativity*, published in 1915. Einstein's theory explains gravity in terms of the curvature of space (or more accurately, *spacetime*) around a massive object, such as the sun or a planet. This spacetime curvature is represented visually in Figure 2.29. Fascinatingly, since Einstein's theory is about curving space, the theory predicts that even phenomena without mass, such as rays of light, are affected by gravity. Einstein noticed this and made the stunning prediction in 1917 that starlight bends as it travels through space when it passes near a massive object such as the sun. He formed this hypothesis, including the amount light bends, based on his general theory of relativity, which was based *completely on mathematics*. What do you think about that? It practically leaves me speechless.

Einstein became instantly world famous in 1919 when his prediction was confirmed. To test this hypothesis, Einstein proposed photographing the stars we see near the sun during a solar eclipse. This has to be done during an eclipse because looking at the sky while the sun is nearby means it is broad daylight and we aren't able to see the stars. Einstein predicted that the apparent position of the stars shifts a tiny amount relative to where they appear when the sun is not near the path of the starlight. British scientist Sir Arthur Eddington commissioned two teams of photographers to photograph the stars during the solar eclipse of 1919. After analyzing their photographic plates (one of which is shown on the opening page of Chapter 1), they found the starlight shifted by exactly the amount Einstein said it would. Talk about sudden fame—Einstein became the instant

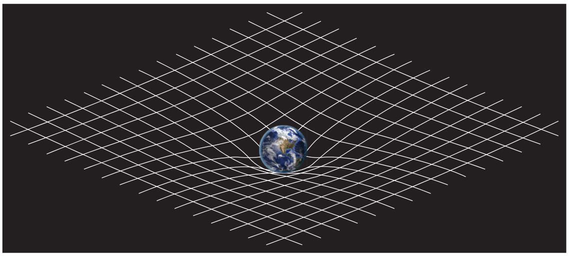


Figure 2.29. A visual representation of the curvature of spacetime around the earth.

global rock star of physics when this happened! (And his puppy dog eyes contributed even more to his popularity!)

Just as Kepler's laws were superseded by Newton's laws and can be derived from Newton's laws, Newton's law of universal gravitation was superseded by Einstein's general theory of relativity and can be derived from general relativity. Einstein believed that his own theories would some day be superseded by an even more all-encompassing theory, but so far (after 103 years) that has not happened. The general theory of relativity remains today the reigning champion theory of gravity, our best understanding of how gravity works, and one of the most important theories in 20th- and 21st-century physics.

Chapter 2 Exercises

Unit Conversions

It is time for you to get busy learning the metric prefixes and unit conversion factors in Appendix A. Perform the following unit conversions, showing all your work in detail. (Showing just the answers is not adequate; show all the conversion factors involved in the conversion for each problem.) Check your work against the answer key on the next page. Where possible, express your results in both standard notation and scientific notation, using the correct number of significant digits. For the first 20 problems, use the standard method of multiplying conversion factors. The last problem requires an extra step that I think you can figure out.

	Convert this Quantity	Into these Units
1	1,750 meters (m)	feet (ft)
2	3.54 grams (g)	kilograms (kg)
3	41.11 milliliters (mL)	liters (L)
4	7×10^8 m (radius of the sun)	miles (mi)
5	1.5499×10^{-12} millimeters (mm)	m
6	750 cubic centimeters (cm^3 or cc) (size of the engine in my old motorcycle)	m^3
7	2.9979×10^8 meters/second (m/s) (speed of light)	ft/s
8	168 hours (hr) (one week)	s
9	5,570 kilograms/cubic meter (kg/m^3) (average density of the earth)	g/cm^3
10	45 gallons per second (gps) (flow rate of Mississippi River at the source)	m^3/minute (m^3/min)
11	600,000 cubic feet/second (ft^3/s) (flow rate of Mississippi River at New Orleans)	liters/hour (L/hr)
12	5,200 mL (volume of blood in a typical man's body)	m^3

	Convert this Quantity	Into these Units
13	5.65×10^2 mm^2 (area of a postage stamp)	square inches (in^2)
14	32.16 ft/s^2 (acceleration of gravity, or one "g")	m/s^2
15	5,001 µg/s	kg/min
16	4.771 g/mL	kg/m^3
17	13.6 g/cm^3 (density of mercury)	mg/m^3
18	93,000,000 mi (distance from earth to the sun)	cm
19	65 miles per hour (mph)	m/s
20	633 nanometers (nm) (wavelength of light from a red laser)	in
21	5.015% of the speed of light	mph

Answers

(A dash indicates that it is either silly or incorrect to write the answer that way, so I didn't: silly because there are simply too many zeros, or no zeros at all; incorrect because we are unable to express the result that way and still show the correct number of significant digits.)

	Standard Notation	Scientific Notation
1	5,740 ft	5.74×10^3 ft
2	0.00354 kg	3.54×10^{-3} kg
3	0.04111 L	4.111×10^{-2} L
4	400,000 mi	4×10^5 mi
5	–	1.5499×10^{-15} m
6	0.00075 m^3	7.5×10^{-4} m^3
7	983,560,000 ft/s	9.8356×10^8 ft/s
8	605,000 s	6.05×10^5 s
9	5.57 g/cm^3	–
10	–	1.0×10^1 m^3/min
11	60,000,000,000 L/hr	6×10^{10} L/hr
12	0.0052 m^3	5.2×10^{-3} m^3
13	0.876 in^2	8.76×10^{-1} in^2
14	9.802 m/s^2	–
15	0.0003001 kg/min	3.001×10^{-4} kg/min
16	4,771 kg/m^3	4.771×10^3 kg/m^3

	Standard Notation	Scientific Notation
17	13,600,000,000 mg/m^3	1.36×10^{10} mg/m^3
18	–	1.5×10^{13} cm
19	29 m/s	2.9×10^{1} m/s
20	0.0000249 in	2.49×10^{-5} in
21	33,700,000 mph	3.37×10^{7} mph

Motion Exercises

1. A train travels 25.1 miles in 0.50 hr. Calculate the velocity of the train.
2. Convert your answer from the previous problem to km/hr.
3. How far can you walk in 4.25 hours if you keep up a steady pace of 5.0000 km/hr? State your answer in km.
4. For the previous problem, how far is this in miles?
5. On the German autobahn there is no speed limit and in good weather many cars travel at velocities exceeding 150.0 mi/hr. How fast is this in km/hr?
6. Referring again to the previous question, how long does it take a car at this velocity to travel 10.0 miles? State your answer in minutes.
7. An object travels 3.0 km at a constant velocity in 1 hr 20.0 min. Calculate the object's velocity and state your answer in m/s.
8. A car starts from rest and accelerates to 45 mi/hr in 36 s. Calculate the car's acceleration and state your answer in m/s^2.
9. A rocket traveling at 31 m/s fires its retro-rockets, generating a negative acceleration (it is slowing down). The rockets are fired for 17 s and afterwards the rocket is traveling at 22 m/s. What is the rocket's acceleration?
10. A person is sitting in a car watching a traffic light. The light is 14.5 m away. When the light changes color, how long does it take the new color of light to travel to the driver so that he can see it? State your answer in nanoseconds. (The speed of light in a vacuum or air, *c*, is one of the physical constants listed in Appendix A that you need to know.)
11. A proton is uniformly accelerated from rest to 80.0% of the speed of light in 18 hours, 6 minutes, 45 seconds. What is the acceleration of the proton?
12. A space ship travels 8.96×10^9 km at 3.45×10^5 m/s. How long does this trip take? Convert your answer from seconds to days.
13. An electron experiences an acceleration of 5.556×10^6 cm/s^2 for a period of 45 ms. If the electron is initially at rest, what is its final velocity?
14. A space ship is traveling at a velocity of 4.005×10^3 m/s when it switches on its rockets. The rockets accelerate the ship at 23.1 m/s^2 for a period of 13.5 s. What is the final velocity of the rocket?
15. A more precise value for *c* (the speed of light) than the value given in Appendix A is 2.9979×10^8 m/s. Use this value for this problem. On a particular day the earth

is 1.4965×10^8 km from the sun. If on this day a solar flare suddenly occurs on the sun, how long does it take an observer on the earth to see it? State your answer in minutes.

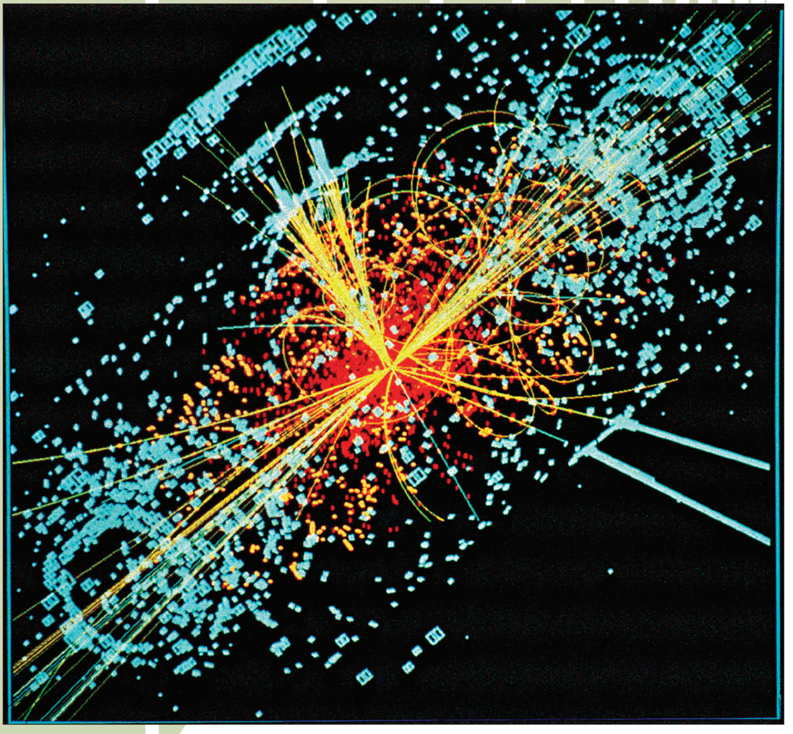
Answers

1. 22 m/s	2. 79 km/hr	3. 21.3 km	4. 13.2 mi
5. 241.4 km/hr	6. 4.00 min	7. 0.63 m/s	8. 0.56 m/s^2
9. –0.53 m/s^2	10. 48.3 ns	11. 3,680 m/s^2	12. 301 days
13. 2.5×10^3 m/s	14. 4.32×10^3 m/s	15. 8.3197 min	

Ptolemaic Model and Copernican Revolution Study Questions

1. Make a list of all the regions in the Ptolemaic Model in their correct order. (There are 10 of them and the first nine are called spheres.) For each of the last three regions write a brief description of the meaning of the name.
2. Describe why some theologians in the 16th century were strongly opposed to Copernicus' heliocentric theory.
3. State six features of the Ptolemaic model other than the spheres.
4. Describe Copernicus' model of the heavens.
5. What are some of the "proofs" people used in arguing that there is no way that the earth rotates on an axis?
6. For what reason did Copernicus decide to keep his theory private?
7. Write a description of the two key observations Tycho made (including dates) that challenged the Ptolemaic system.
8. Briefly describe the cosmological model put forward by Tycho.
9. State Kepler's first law of planetary motion.
10. This is a bit difficult, but explain retrograde motion and epicycles as well as you can in a few sentences.
11. Explain the two main mistakes individuals made that led to Galileo's trial.
12. Explain the actual cause of Galileo's trial and the results of that trial.
13. Describe why Pope John Paul II commended Galileo in 1992.
14. Distinguish between Newton's and Einstein's theories of gravitation. According to each of these two geniuses, what is the cause of gravity and what are the effects of gravity?
15. The theories reviewed in this chapter suggest that the universe possesses a very deep mathematical structure. What does this structure indicate about where the universe came from?
16. Describe some of Kepler's scientific achievements, aside from his laws of planetary motion.

CHAPTER 4
Energy



Large Hadron Collider

In the Large Hadron Collider (LHC) at CERN in Switzerland (see box on page 12), protons are accelerated and collided at extremely high energies. The purpose of these collisions is to help scientists discover more about the fundamental structure of matter. Theory predicts the existence of a particle called the Higgs Boson. The image above is a computer simulation of a Higgs detection event inside the CMS detector at the LHC. The CMS website states, "The lines represent the possible paths of particles produced by the proton-proton collision in the detector while the energy these particles deposit is shown in blue."

As you know, the speed of light is 300,000,000 m/s. To generate the energy needed to observe the Higgs Boson, protons are accelerated to a speed that is only 3 m/s slower than the speed of light! At this speed, the protons only require 90 μs (0.000090 s) to travel 17 miles around the main underground tunnel of the LHC. This huge kinetic energy is way beyond the energy produced by any other particle accelerator yet constructed.

OBJECTIVES

Memorize and learn how to use these equations:

$$E_G = mgh \qquad E_K = \tfrac{1}{2}mv^2 \qquad v = \sqrt{\frac{2E_K}{m}} \qquad W = Fd$$

After studying this chapter and completing the exercises, students will be able to do each of the following tasks, using supporting terms and principles as necessary:

1. State the law of conservation of energy.
2. Describe how energy can be changed from one form to another, including:
   a. different forms of mechanical energy (kinetic, gravitational potential, elastic potential)
   b. chemical potential energy
   c. electrical energy
   d. elastic potential energy
   e. thermal energy
   f. electromagnetic radiation
   g. nuclear energy
   h. acoustic energy
3. Briefly define each of the types of energy listed above.
4. Describe two processes by which energy can be transferred from one object to another (work and heat), and the conditions that must be present for the energy transfer to occur.
5. Describe in detail how energy from the sun is converted through various forms to end up as energy in our bodies, as energy used to run appliances in our homes, or as energy used to power machines in industry.
6. Explain why the efficiency of any energy conversion process is less than 100%.
7. Calculate kinetic energy, gravitational potential energy, work, heights, velocities, and masses from given information using correct units of measure.
8. Define friction.
9. Using the pendulum as a case in point, explain the behavior of ideal and actual systems in terms of mechanical energy.
10. Explain how friction affects the total energy present in a mechanical system.

4.1 What is Energy?

4.1.1 Defining Energy

Defining energy is tricky. Dictionaries usually say, "the capacity to do mechanical work," which is not particularly helpful. Actually, there is no definition for energy that gets at what it actually *is*, so I will not try to define it. We are just going to accept that energy exists in the universe, it was put there by God when he made the universe, and it exists in many different forms. It is fairly obvious that a bullet traveling at 2,000 ft/s has more energy than a bullet at rest. This is why the high speed bullet can kill but the bullet at rest cannot. This study is mainly about tracking energy as it changes from one form to another, and calculating the quantities of three particular forms of energy.

4.1.2 The Law of Conservation of Energy

The *law of conservation of energy* is as follows:

Energy can be neither created nor destroyed, only changed in form.

Energy can be in many different forms in different types of substances, such as in the molecules of gasoline, in the waves of a beam of light, in heat radiating through space, in moving objects, in compressed springs, or in objects lifted vertically on earth. As different physical processes occur—such as digesting food, throwing a ball, operating a machine, heating due to friction, or accelerating a race car—energy in one form is being converted into some other form. Energy might be in one form in one place, such as in the chemical potential energy in the muscles of your arm, and be converted through a process such as throwing a ball to become energy in another form in another place, such as kinetic energy in the ball.

4.1.3 Mass-Energy Equivalence

In 1905, Albert Einstein published his now-famous equation, $E = mc^2$. The E and m in this equation represent energy and mass; c represents the speed of light. With this equation, Einstein theorized that mass and energy are really just different forms of the same thing. That is, all mass has associated with it an equivalent amount of energy (given by $E = mc^2$), and vice versa. This theory of mass-energy equivalence is now considered to be a fundamental property of the universe.

The reason I mention mass-energy equivalence here is that since mass is a form of energy, matter must be taken into consideration for a completely accurate statement of the law of conservation of energy. In nuclear reactions, such as take place in the sun (fusion) or in nuclear power plants (fission), quantities of matter are converted completely into energy. Einstein's equation $E = mc^2$ also gives the amount of energy that appears when a quantity of matter in converted to energy in one of these nuclear processes. Thus, to be completely accurate, we need to state that the law of conservation of energy includes all mass as well, as one of the forms energy can take. Let's restate the conservation law with this in mind: "*mass-energy can be neither created nor destroyed, only changed in form*."

Most of the problems we encounter in physics and chemistry don't involve nuclear reactions (thankfully). This means that for most purposes, we can consider the common forms of energy listed below without worrying about the complicated issue of mass-energy equivalence.

4.2 Energy Transformations

4.2.1 Forms of Energy

Here are some common forms energy can take:

Gravitational Potential Energy This is the energy an object possesses because it has been lifted up in a gravitational field. If such an object is released and allowed to fall, the gravitational potential energy converts into kinetic energy. The term *potential* in the name of this form of energy indicates that the energy is stored and converts into another form of energy when released. There are other forms of energy listed below that use this term for the same reason.

Kinetic Energy This is the energy an object possesses because it is in motion. The faster an object is moving, the more kinetic energy the object has.

Electromagnetic Radiation This is the energy in electromagnetic waves traveling through space, or through media such as air or glass. This type of energy includes all forms of light, as well as radio waves, microwaves, and a number of other kinds of radiation. We study electromagnetic waves in some detail in a later chapter.

Chemical Potential Energy This energy is in the chemical bonds of molecules. In the case of substances that burn, the chemical potential energy in the molecules is released in large quantities as heat and light when the substance is burned, making these substances useful as fuel.

Electrical Energy This is energy flowing in electrical conductors, such as from a power station to your house to power your appliances.

Thermal Energy This is the energy a substance possesses due to being heated. We examine thermal energy more closely in Chapters 6 and 7.

Elastic Potential Energy This is the energy contained in any object that has been stretched (such as a bungee cord or a hunter's bow) or compressed (such as a spring).

Nuclear Energy This is energy released from the nuclear processes of fission (when the nuclei of atoms are split apart) or fusion (when atomic nuclei are fused together). As I mention in the previous section, these processes convert mass into energy.

Acoustic (Sound) Energy This is the energy carried in sound waves, such as from a person's voice, the speakers in a sound system, or the noise of an explosion. Since sound waves are carried by moving air molecules, this is really a special form of kinetic energy.

4.2.2 Energy Transfer

Two more important energy-related terms are those associated with the process of energy being transferred from one place, substance, or object to another. These two terms are:

Work Work is a mechanical process by which energy is transferred from one object to another. Objects do not possess work like they do other forms of energy. Instead, one object "does work" on another object by applying a force to it and moving it a certain distance. When one object does work on another, energy is transferred from the first object to the second. We study work in more detail later in the chapter.

Heat The term *heat* is used as a general description of energy in transit, flowing by various means from a hot substance to a cooler substance when a difference in temperature is present. We study heat and the three ways it flows in more detail in Chapter 7. As with work, substances do not possess heat. What substances do possess is kinetic energy in their moving atoms, and we refer to this energy as the *internal energy* of the substance.

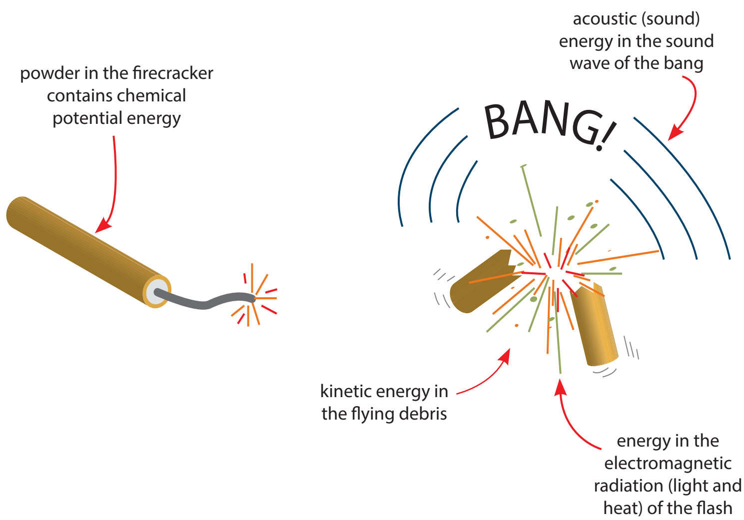


Figure 4.1. Chemical potential energy in a firecracker is converted into other forms of energy when the firecracker explodes.

Let's look at a common example of energy changing from one form into others. We all know what happens when a person lights a firecracker. (It explodes!) What forms of energy are present during the explosion, and where did all this energy come from? As illustrated in Figure 4.1, the energy released in the explosion is the chemical potential energy in the molecular bonds of the chemicals inside the firecracker. When these chemicals burn, they release a lot of energy. And as you already know, an exploding firecracker gives off a flash of light and heat (both are forms of electromagnetic radiation), a loud bang, and the fragments of the firecracker are blown all over the place. Thus, the chemical potential energy in the powder inside the firecracker is converted into several different kinds of energy during the explosion.

Now consider how the law of conservation of energy applies to this explosion. All the energy present in the chemicals before the explosion is still present in various forms after the firecracker explodes. This is what "conservation" of energy means. We can represent the conservation of energy in a sort of equation like this:

chemical potential energy in the firecracker	=	acoustic energy in the bang	+	kinetic energy of flying debris	+	energy in light and heat

We do not perform any calculations this complex in this course. But later in this chapter, we begin using the principle of conservation of energy to solve problems involving three of the forms of energy we have seen so far.

4.2.3 The "Energy Trail"

Much of the energy we depend on here on earth comes to us from the sun. As we track the forms this energy takes in its journey from the sun to, say, the energy in our bodies, we might call this the "energy trail." (This way we can have fun yelling *Yee-Haw!* while we are studying this. Ask your teacher for a demonstration.) We now follow the energy trail beginning with the sun, through different processes of conversion, and arriving at different places where energy is commonly used.

The sun's energy is produced by fusion reactions as the nuclei of hydrogen atoms "fuse" or stick together to form helium. This is so hard to do that we have not yet succeeded in doing it here on earth in a controlled way. However, we have succeeded in doing it in an un-

Do You Know ... *What is dark energy?*

When Einstein first developed his equations for the general theory of relativity (published in 1915), the equations implied that the universe must be either expanding or contracting. At the time, the prevailing view was that the universe was doing neither, so Einstein put a fudge factor in the equations to keep the universe static.

Just a few years later, astronomer Edwin Hubble made observations that enabled Belgian Georges Lemaître, a physicist and Catholic priest, to conclude that the universe was expanding, just like Einstein's original equations implied! So Einstein took the fudge factor out of the equations and said that putting it in was "the biggest blunder of his life."

In the 1990s, astronomers discovered that the expansion of the universe is *speeding up*—the expansion of the universe is *accelerating*. This seems impossible, because the gravitational attraction of the galaxies pulling on each other should be slowing the expansion rate down. In a classic illustration of how the Cycle of Scientific Enterprise works, no known theory was able to account for the cause of the acceleration, so scientists had to get busy theorizing about this mystery.

At present, the most accepted hypothesis for the cause of the acceleration is the presence of an unknown form of energy called dark energy that permeates all of space. Calculations indicate that on the basis of mass-energy equivalence, 68% of the energy in the universe is dark energy, 27% of the energy is in the form of dark matter (see the box on page 8), and only 5% of the energy is in the form of ordinary matter.

controlled way. Fusion is the same nuclear reaction as the main reaction in a thermonuclear bomb. A thermonuclear explosion is an uncontrolled nuclear fusion reaction.

When referring to this energy being produced in the sun, we can simply call it nuclear energy. The energy leaves the sun as electromagnetic radiation, a different form of energy, and travels through space to us. When this energy arrives at earth, most of it warms the ground, oceans, and atmosphere. This is very important for stabilizing the earth's climate and making earth habitable, but unless we collect the energy in a solar collector of some kind we are not able to use this energy directly.

However, some of the electromagnetic radiation streaming from the sun is captured by plants and converted into chemical potential energy in the molecules in the plants through the process of photosynthesis. These plants eventually become the foods we eat or the fuels we burn. Current theory holds that in ancient eras in the earth's history, many vast forests were buried and the plant matter was converted underground into what we now call "fossil fuels" (petroleum, coal, and natural gas). Some fuels come from living plants too, such as firewood from trees and automotive alcohol (ethanol) from corn. The energy in the molecules of these fuels is chemical potential energy that is converted into heat energy when the fuels are burned.

Your task is to describe the energy conversions each step of the way from the sun all the way to your breakfast cereal or your computer. Tables 4.1 through 4.4 illustrate a few examples of following the energy from the sun to different places it can end up here on earth. When asked to outline one of these pathways in the "energy trail," always list two things for each step of the way: (1) Where the energy is, and (2) what form the energy is in.

Where is the energy?	The Sun	Electro-magnetic waves in space	Plants on earth	Breakfast cereal	Muscles in the human body	Stretched bow	Flying arrow
What form is the energy in?	Nuclear energy	Electro-magnetic radiation	Chemical potential energy	Chemical potential energy	Chemical potential energy	Elastic potential energy	Kinetic energy

Table 4.1. Energy transformations from the sun to a flying arrow, assuming the archer was on a vegetarian diet.

Where is the energy?	The Sun	Electro-magnetic waves in space	Plants on earth	Chicken feed	Muscles in the bodies of chickens	Muscles in the human body	Moving kid on skate-board
What form is the energy in?	Nuclear energy	Electro-magnetic radiation	Chemical potential energy	Chemical potential energy	Chemical potential energy	Chemical potential energy	Kinetic energy

Table 4.2. Energy transformations from the sun to a kid on a skateboard, assuming the kid was eating chicken.

Where is the energy?	The Sun	Electro-magnetic waves in space	Plants on earth	Fossil fuel (crude oil, coal, natu-ral gas)	Spin-ning gas turbine generator at power station	Wires from the power station to your house	Heat from the coils in the toaster
What form is the energy in?	Nuclear energy	Electro-magnetic radiation	Chemical potential energy	Chemical potential energy	Kinetic energy	Electrical energy	Electro-magnetic radiation

Table 4.3. Energy transformations from the sun to the heat from a toaster in your house.

Where is the energy?	The Sun	Electro-magnetic waves in space	Plants on earth	Fossil fuel (coal)	Heat from burning coal	Steam in the boiler	Moving train
What form is the energy in?	Nuclear energy	Electro-magnetic radiation	Chemical potential energy	Chemical potential energy	Heat	Thermal energy	Kinetic energy

Table 4.4. Energy transformations from the sun to a moving steam locomotive.

4.2.4 The Effect of Friction on a Mechanical System

You probably already have a feel for what people mean by the term *friction*. Friction is a force present any time one object or material comes in contact with another object or material. The effect of friction is to oppose any relative motion between the two objects in contact. The cause of friction is rather complicated, but down at the atomic level friction has to do with the electrical attractions and repulsions between the charged particles in the atoms of the objects.

Friction makes it harder for one object to slide on top of another, which is good if you are talking about the friction between the tires of a car and the pavement. If there were no friction, cars could not start or stop or steer. (You may experience this physically if you are ever in the undesirable position of trying to drive a car on ice.) Friction is also very nice to have around any time one is attempting to grab something or clamp something. Without friction, things would just slip through our fingers. But friction is undesirable when the goal is to design the parts of a machine so they slide smoothly against one another without wear or damage to the machine. And, of course, there is friction when an object moves through the air. This friction is usually called air resistance or *drag*.

In this course, we are not considering friction in the calculations we do. However, in all real mechanical systems friction plays a significant role. Friction is caused when parts of the system rub against each other or when parts of the system move through a fluid such as air or water. Just as when you rub your hands together on a cold day, friction always results in heating. When the parts of a mechanical system such as a machine get warm from friction, heat flows from the warm parts into the cooler surrounding environment. (We will look more at how this happens in Chapter 7.) This heat energy flowing out of the system is energy that used to be in the system.

When heat energy flows out of a system due to friction, the law of conservation of energy still applies: no energy is created or destroyed. However, the energy remaining in the system is reduced by the amount of energy that flows out of the system due to heating from friction. A scientist or engineer may refer to energy "lost" due to friction. This does not mean the energy is destroyed or ceases to exist, only that it flows out of the system as heat and is no longer available as energy in the system. The net effect, of course, is that things slow down as energy gradually leaves the system as heat due to friction.

4.2.5 Energy "Losses" and Efficiency

For all the different forms of energy we have considered, there are many different kinds of processes that might be involved in converting energy from one form to another. Combustion is a process that converts chemical potential energy into heat. Photosynthesis converts electromagnetic energy from the sun into chemical potential energy in the cells of plants. The Industrial Revolution began when humans began learning how to design machines and systems to convert energy from various forms found in nature into forms that can be harnessed to do useful work for us.

Let's consider some process like this, such as an engine in a car converting the chemical potential energy in the gasoline into kinetic energy in the moving car. One of the facts of life on earth is that it is theoretically impossible for a conversion process to capture all the energy involved and convert it to a form that can do useful work. Whether we want it to or not, some of the energy always converts to heat, which radiates out into the environment. The laws of thermodynamics state that this is always the case.

This situation is represented in Figure 4.2. It is common to speak of the energy converted into heat as "lost." Keeping the law of conservation of energy in mind, it should be clear that what we mean by this is not that the energy ceases to exist, only that the energy escapes into the environment where it is no longer available to us in a usable form. It is lost from the system, not from the universe.

The *efficiency* of an energy conversion process is the ratio of the usable energy coming out of the process to the energy that goes into the process:

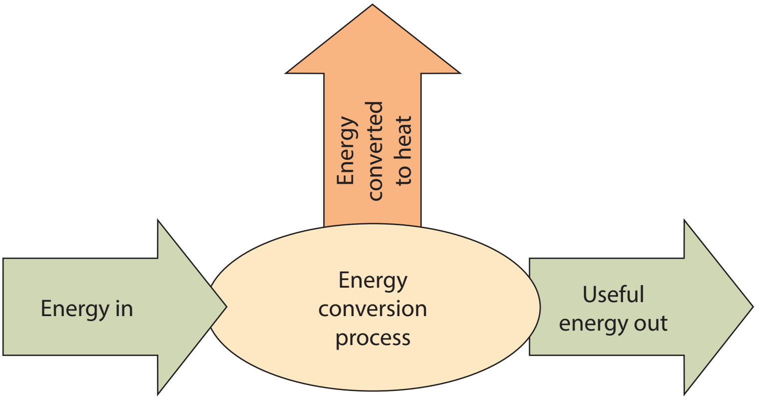


Figure 4.2. In any energy conversion process, some energy is converted to heat that is not available to do useful work.

$$Efficiency = \frac{\text{useful energy out}}{\text{energy in}} \times 100\%$$

Since some energy is always lost as heat, the efficiencies of our machines are always less than 100%. As physical examples, the efficiency of typical automobile engines is only around 15%, which means that 85% of the energy in the fuel is not used to propel the car. (That's a lot of lost energy.) Solar cells convert electromagnetic radiation from the sun into electricity. At present, the highest efficiency realized with these technologies is around 25%. The overall efficiency of the new electric cars is around 20–25%. This figure may seem low, but there are a lot of losses in generating the electrical power at a power station and transporting the power to the homes where people charge up the batteries in their electric cars.

4.3 Calculations with Energy

4.3.1 Gravitational Potential Energy and Kinetic Energy

Two important forms energy can take in mechanical systems are gravitational potential energy, E_G, and kinetic energy, E_K. The gravitational potential energy an object possesses depends on how high up it is and the kinetic energy of an object depends on how fast it is moving. Both also depend on the object's mass. Gravitational potential energy is calculated as

$$E_G = mgh$$

where E_G is energy in joules (J), m is the mass (kg), g = 9.80 m/s^2, and h is the height (m).

Notice that if you know how much gravitational potential energy an object has and its mass, you can solve this equation for h to find out how high the object is above the ground. Simply divide both sides of the equation by mg and you have

$$h = \frac{E_G}{mg}$$

Notice also that the gravitational potential energy of an object is directly proportional to its height. If the height of an object increases by 50%, the gravitational potential energy of the object also increases by 50%. When calculating gravitational potential energy, the energy you calculate always depends on the location you choose to use as your zero reference for the height. This zero reference might be sea level, or the ground, or the floor of your classroom, or a table top. It doesn't matter, because the E_G an object has is always relative to where $h = 0$ is. Usually, the most logical and convenient location for $h = 0$ is clear from the context.

The equation for gravitational potential energy gives us another example of a derived MKS unit, the joule, for quantities of energy. Multiplying the units together for the terms on the right side of the E_G equation, we can see that a joule is made up of primary units as follows:

$$1\text{ J} = 1\text{ kg}\cdot\frac{\text{m}}{\text{s}^2}\cdot\text{m} = 1\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

You might compare this to the units described at the bottom of page 22.

▼ Example 4.1

A golf ball has a mass of 45.9 g. While climbing a tree near a golf course, little Janie finds a golf ball stuck in a branch 9.5 ft above the ground. What is the gravitational potential energy of the golf ball at that height?

Start by writing the givens and doing the unit conversions to get all quantities into MKS units, keeping one extra significant digit in your intermediate calculations.

$$m = 45.9\text{ g}\cdot\frac{1\text{ kg}}{1000\text{ g}} = 0.0459\text{ kg}$$

$$h = 9.5\text{ ft}\cdot\frac{0.3048\text{ m}}{\text{ft}} = 2.90\text{ m}$$

$$E_G = ?$$

Now write the equation and complete the problem.

$$E_G = mgh = 0.0459\text{ kg}\cdot 9.80\ \frac{\text{m}}{\text{s}^2}\cdot 2.90\text{ m} = 1.30\text{ J}$$

These calculations all contain one extra significant digit. The given height only has two significant digits, so now we round our final result to two digits.

$$E_G = 1.3\text{ J}$$

▲

▼ Example 4.2

An ant carries a grain of sugar up the side of a building to its nest on the roof. The mass of the grain of sugar is 0.0356 μg. After it has been carried to the roof, the E_G in the grain of sugar is 1.91 nJ. How high is the ant nest?

Write the givens and do the unit conversions.

$$m = 0.0356\ \mu\text{g} \cdot \frac{1\ \text{g}}{10^6\ \mu\text{g}} \cdot \frac{1\ \text{kg}}{1000\ \text{g}} = 3.56 \times 10^{-11}\ \text{kg}$$

$$E_G = 1.91\ \text{nJ} \cdot \frac{1\ \text{J}}{10^9\ \text{nJ}} = 1.91 \times 10^{-9}\ \text{J}$$

$$h = ?$$

Now write the equation, solve for h, and compute the result.

$$E_G = mgh$$

$$h = \frac{E_G}{mg} = \frac{1.91 \times 10^{-9}\ \text{J}}{3.56 \times 10^{-11}\ \text{kg} \cdot 9.80\ \frac{\text{m}}{\text{s}^2}} = 5.47\ \text{m}$$

Every value in this computation has three significant digits, as does this result, so the problem is complete.

▲

Now we look at another important form of energy, kinetic energy. Kinetic energy is one of the most important concepts in physics because it relates to many other concepts. Kinetic energy is calculated as

$$E_K = \tfrac{1}{2} m v^2$$

where E_K is the kinetic energy in joules (J), m is the mass (kg), and v is the velocity (m/s). The units for kinetic energy are joules, just as with all other forms of energy. Kinetic energy is proportional to the mass of an object and to the square of the object's velocity.

Notice that if you know how much kinetic energy an object has and its mass, you can solve this equation for v to find out how fast the object is moving. Since the algebra to do this may be unfamiliar to students in this course, you may want to just go ahead and memorize the equation for velocity as a function of kinetic energy. This equation is

$$v = \sqrt{\frac{2E_K}{m}}$$

▼ Example 4.3

An electron with a mass of 9.11×10^{-28} g is traveling at 1.066% the speed of light. Determine the amount of kinetic energy the electron has and state your result in nJ.

Start by writing the givens and doing the unit conversions. To obtain the electron's velocity, we must multiply the speed of light (from Appendix A) by 0.01066.

$$m = 9.11 \times 10^{-28}\ \text{g} \cdot \frac{1\ \text{kg}}{1000\ \text{g}} = 9.11 \times 10^{-31}\ \text{kg}$$

$$v = 0.01066 \cdot 3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}} = 3.198 \times 10^{6}\ \frac{\text{m}}{\text{s}}$$

$$E_K = ?$$

Now compute the kinetic energy.

$$E_K = \tfrac{1}{2} m v^2 = 0.5 \cdot 9.11 \times 10^{-31}\ \text{kg} \cdot \left(3.198 \times 10^{6}\ \frac{\text{m}}{\text{s}}\right)^2 = 4.658 \times 10^{-18}\ \text{J}$$

The problem statement requires the result to be in units of nanojoules (nJ), so perform this conversion.

$$4.658 \times 10^{-18}\ \text{J} \cdot \frac{10^{9}\ \text{nJ}}{\text{J}} = 4.658 \times 10^{-9}\ \text{nJ}$$

Both the mass and the speed of light values have three significant digits, so rounding this result to three significant digits gives

$$E_K = 4.66 \times 10^{-9}\ \text{nJ}$$

▲

▼ Example 4.4

A kid fires a plastic dart from a dart gun. The mass of the dart is 21.15 g and its kinetic energy is 0.3688 J when it flies out the dart gun. Determine the velocity of the dart.

Write the givens and do the unit conversions.

$$m = 21.15\ \text{g} \cdot \frac{1\ \text{kg}}{1000\ \text{g}} = 0.02115\ \text{kg}$$

$$E_K = 0.3688\ \text{J}$$

$$v = ?$$

Now complete the problem using the memorized velocity equation.

$$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2 \cdot 0.3688\ \text{J}}{0.02115\ \text{kg}}} = 5.905\ \frac{\text{m}}{\text{s}}$$

Both the mass and the kinetic energy values have four significant digits, so this result is rounded to four significant digits.

▲

4.3.2 Work

The way an object acquires kinetic energy or gravitational potential energy is that another object or person or machine does *work* on it. Work is the way mechanical energy is transferred from one machine or object to another. Work is a form of energy, but objects don't possess work. Work is the process by which energy is transferred from one mechanical system to another. Work is defined as the energy it takes to push an object with a certain (constant) force over a certain distance. Work is calculated as

$$W = Fd$$

where W is the work done on the object in joules (J), F is the force on the object (N), and d is the distance the object moves (m).

Notice from this equation that since work is energy, the units here come out to be

$$1\ \text{J} = 1\ \text{N} \cdot \text{m}$$

Let's take a moment to convince ourselves that the units here are the same as the units described above right after the E_G equation. The work equation says that joules are equal to newtons times meters. A newton is a force, and we know from Newton's second law of motion that force equals mass times acceleration, or $F = ma$. If we multiply all these units together we have

$$1\ \text{J} = 1\ \text{N} \cdot \text{m} = 1\ \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m} = 1\ \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

These units are indeed the same as the units we worked out for gravitational potential energy a few pages back.

The concept of work is the basic principle governing how energy is transferred from one device to another in a *mechanical system*. For example, as depicted in Figure 4.3, if an electric motor is used to lift a piece of equipment, the motor must reel in a certain length, L, of steel cable, and it must pull on the cable with a certain force, F, while doing so. The pulling force times the length of cable is the amount of work done by the motor. And where does this work energy supplied by the motor go? Assuming 100% efficiency in the lifting motor and cables (and electric motors have very high efficiencies, so this is not a bad approximation), the energy all goes into the gravitational potential energy acquired by the piece of equipment being lifted. In actuality, since the efficiency of all systems is less than 100%, some of the energy leaves the system as heat. In the end, the gravitational potential energy of the lifted piece of equipment does not quite represent all of the energy the electric motor has to supply.

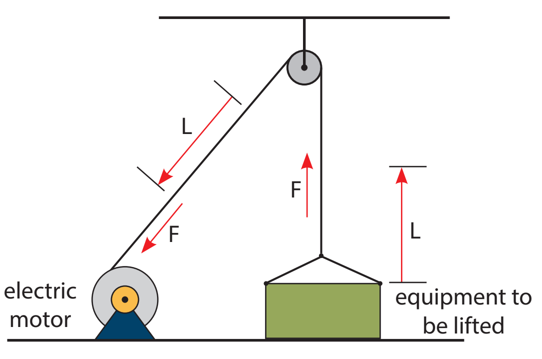


Figure 4.3. An electric motor raising an object to a height L by means of force F pulling on the cable.

We see here that work and conservation of energy are very closely related. As another example, if a man pushes a kid on a bicycle over a short distance to get the kid going, the man delivers energy to the kid on the bicycle equal to the pushing force times the distance pushed. Ignoring friction for now, that work energy from the man is now in the kinetic energy of the kid on the bicycle.

There are two more important details to note about work. First, the equation for work, $W = Fd$, requires that the force applied to an object and the distance the object travels must lie in the same direction. As depicted in Figure 4.4, if a person lifts a bucket of water, then work is done on the bucket of water. The force is applied vertically and the bucket moves vertically, so the work done to lift a bucket of water is the force required to lift it, its weight, times the distance it is lifted. But a person carrying a bucket of water down the road is *not doing any work on the bucket*. This is because the force on the bucket to hold it up is vertical, but the distance the bucket is moving is horizontal. These two forces do not point in the same direction. In fact, they are at right angles to one another and no work is done on the bucket of water.

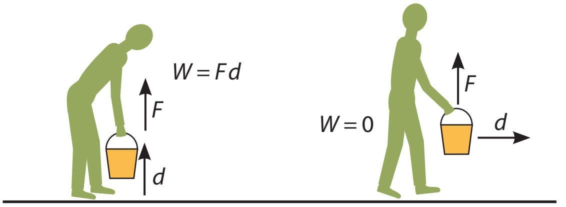


Figure 4.4. In raising the bucket (left) work is done on the bucket. In moving the bucket horizontally, no work is done on the bucket.

The second detail is foreshadowed in the previous paragraph. People often say that the force required to lift a bucket is just a little larger than the weight, but this is not correct. If the upward force is at all greater than the weight, then we have a net upward force on the bucket. Recall from Chapter 3 that Newton's second law of motion says that a net force does not just raise the bucket, it *accelerates* the bucket, giving it kinetic energy. But for a bucket to move up with a *constant* speed requires no net force at all, according to the first law of motion. So after a little bump of force to get the bucket moving (which does briefly require a larger force and some energy), the bucket can be lifted to any height with a force equal to the bucket's weight. In physics problems of this kind, we normally just neglect the little bump of force necessary to get the bucket started and assume that the force required to lift the bucket is equal to the weight of the bucket.

So, a handy problem solving tip to keep in mind is this: *The force required to lift an object is equal to its weight*. Recall that you can always calculate the weight of an object from its mass as

$$F_w = mg$$

Do You Know ... What is alpha radiation?

Nuclear radiation is emitted from radioactive substances during the process of nuclear decay. One form of radiation, called *alpha radiation*, consists of alpha particles streaming out of the radioactive substance. These incredibly small and fast moving alpha particles each consist of two protons and two neutrons. They receive their kinetic energy from radioactive atoms as matter in the atom converts into kinetic energy during the process of nuclear decay.

In Chapter 6, we encounter the story of Ernest Rutherford, who used the alpha particles from a compound called radium bromide to explore the structure of the atom. The alpha particles emitted from radium bromide are travelling at 15,000,000 m/s! This is 9,300 miles per second, a speed that is 5% the speed of light!

▼ Example 4.5

An elevator in a skyscraper has a mass of 904.9 kg. Inside the elevator are three people whose masses are 67.8 kg, 55.9 kg, and 75.1 kg. Determine how much work the elevator motor does in lifting this elevator and the people inside it from the ground floor up to the 47th floor, 564 ft above the ground floor. Assume there is no friction and state the result in kJ.

Write the givens and do the unit conversions.

$$m = 904.9 \text{ kg} + 67.8 \text{ kg} + 59.9 \text{ kg} + 75.1 \text{ kg} = 1103.7 \text{ kg}$$

$$d = 564 \text{ ft} \cdot \frac{0.3048 \text{ m}}{\text{ft}} = 171.9 \text{ m}$$

$$W = ?$$

As I write just above, the force required to lift an object is equal to its weight. So next we need to compute the weight of the elevator and the people.

$$F_w = mg = 1103.7 \text{ kg} \cdot 9.80 \frac{\text{m}}{\text{s}^2} = 10{,}820 \text{ N}$$

My calculator has a lot more digits in it than this, but I see that several of the pieces of given information have three significant digits, and I only need one extra digit for intermediate calculations, so I round to four digits.

Now complete the problem.

$$W = Fd = F_w d = 10{,}820 \text{ N} \cdot 171.9 \text{ m} = 1{,}860{,}000 \text{ J}$$

This result is rounded to three significant digits. As a last step, we convert this value to kilojoules, as required by the problem statement.

$$W = 1{,}860{,}000 \text{ J} \cdot \frac{1 \text{ kJ}}{1000 \text{ J}} = 1860 \text{ kJ}$$

▲

4.3.3 Applying Conservation of Energy

When an object is thrown or fired straight up from the ground, it leaves the ground with a certain velocity, and thus a certain amount of E_K. As it goes up, what happens to this E_K? It is converted to E_G, of course, as the object goes higher and higher and goes slower and slower. At the top of its flight, all the energy the object has at the ground in E_K has been converted into E_G. We can use the law of conservation of energy, along with the equations for E_G and E_K, to determine how high the object goes.

The same thing works in reverse. An object at a certain height has E_G. If the object is then released, as it falls the E_G is gradually and continuously converted into E_K. Just before it hits the ground, all the E_G it has at the top has been converted into E_K. We can use the law of conservation of energy, along with the equations for E_G and E_K, to find out how fast the object is going just before it strikes the ground.

In all the problems we do in this course involving conservation of energy, we are ignoring friction. In reality, friction is always present in any so-called mechanical system, such as moving objects or machines. In Section 4.2.4, we considered the effect friction has on mechanical systems, but in our computations we ignore it. Many physical systems can be approximated pretty well even if friction is ignored. In the conservation of energy experiment (the Hot Wheels Experiment), friction is low enough that the experimental velocity you measure should agree fairly well with the prediction.

Let us now look at a simple example of the conservation of energy in action. Figure 4.5 illustrates the application of conservation of energy to a person lifting a bucket and letting it drop. When a person lifts a bucket vertically, the person does work on the bucket. To compute this work, the force to lift the bucket is the weight of the bucket and the distance involved is the height it is lifted, so the work done on the bucket by the person is

$$W = F_w h$$

Since the weight, F_w, is equal to mg, this equation can be written as

$$W = F_w h = mgh$$

Energy is transferred from the person (the chemical potential energy in the person's muscles) to the bucket, and the bucket now has gravitational potential energy equal to

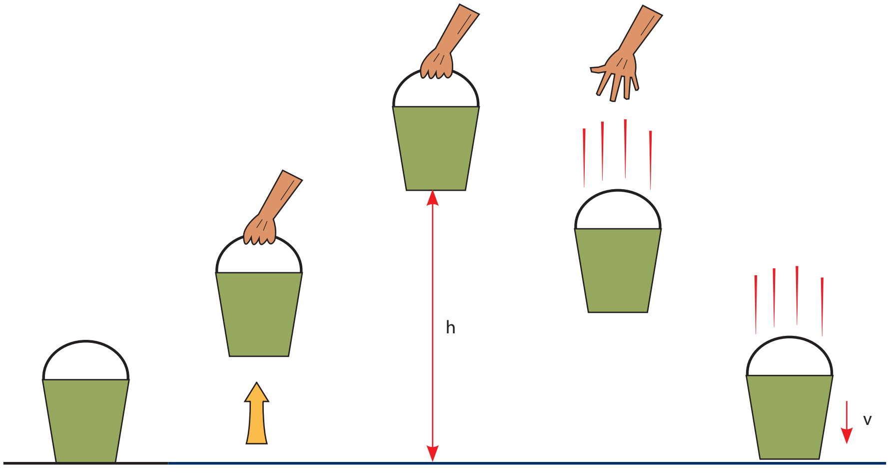


Figure 4.5. Conservation of energy applied to a lifted and falling bucket.

$$E_G = mgh$$

Right here we can see the conservation of energy at work. The work done by the person to lift the bucket is *mgh*. Where did that energy go? It went into the E_G the bucket has at the top, which is *mgh*, the same amount of energy. If the person releases the bucket, then as the bucket falls the gravitational potential energy begins to convert to kinetic energy. At any point as the bucket is falling, energy is conserved, which means the total energy the bucket has is still the same as the energy it has at the top, but some of the energy is in kinetic energy and some of it is in gravitational potential energy. At the instant before the bucket hits the ground, there is no more gravitational potential energy because the height then is zero, so all the energy originally given to the bucket by the work done on it is in the kinetic energy of the bucket.

4.3.4 Conservation of Energy Problems

Now we look at a couple of example problems with objects falling down or flying up. In these kinds of problems, we use the basic principle of conservation of energy to find out how high an object goes or how fast an object is going just before it lands.

A helpful problem solving technique for these kinds of problems is to draw a little diagram for yourself to indicate whether the E_G is converting to E_K ($E_G \rightarrow E_K$), or vice versa ($E_K \rightarrow E_G$). This helps you keep track of what you are doing so you don't become confused. I demonstrate this in the example problems we do below.

One more thing before we do those examples. To help you continue to remember how to calculate an object's mass from its weight, I like to design these problems by giving you the weight instead of the mass that you need. So just first do a separate little problem to obtain the object's mass. Then proceed with the energy calculations. The examples that follow make all these things clear.

▼ Example 4.6

A certain bucket of paint weighs 8.55 lb and is carried up a ladder until it is 4.750 ft above the ground. Sadly, the bucket then falls off the ladder. How fast is the bucket of paint moving just before it hits the ground and makes a colossal mess?

To start, use the weight equation to obtain the mass of the bucket. As always, we first convert the given weight into MKS units.

$$F_w = 8.55\ \text{lb} \cdot \frac{4.45\ \text{N}}{\text{lb}} = 38.05\ \text{N}$$

$$m = ?$$

$$F_w = mg$$

$$m = \frac{F_w}{g} = \frac{38.05\ \text{N}}{9.80\ \frac{\text{m}}{\text{s}^2}} = 3.883\ \text{kg}$$

Notice that I do these calculations with four significant digits. This is because the value for *g* has three significant digits and my intermediate results always have an extra digit before I round off at the end.

Just as a reminder, the beauty of working in the MKS unit system is that when we use MKS units in a calculation, the result always has MKS units. So when I divide newtons (N) by meters per second squared (m/s^2), I don't have to worry about puzzling out any unit issues. I know this calculation gives me a mass, and the MKS unit for mass is the kilogram (kg).

Now that we have the mass, let's write down everything and begin the energy calculation.

$$m = 3.883 \text{ kg}$$

$$h = 4.750 \text{ ft} \cdot \frac{0.3048 \text{ m}}{\text{ft}} = 1.448 \text{ m}$$

$$v = ?$$

Now here is our energy diagram for this problem:

$$E_G \rightarrow E_K$$

This tells me that in this problem, all the E_G we have to begin with converts into E_K as the bucket falls. So I need to calculate the E_G first and then use this amount of energy as the E_K for calculating the velocity.

$$E_G = mgh = 3.883 \text{ kg} \cdot 9.80 \ \frac{\text{m}}{\text{s}^2} \cdot 1.448 \text{ m} = 55.10 \text{ J}$$

Since this gravitational potential energy converts to kinetic energy, we now have

$$E_K = 55.10 \text{ J}$$

Finally, we use this value in the velocity equation to obtain our final result.

$$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2 \cdot 55.10 \text{ J}}{3.883 \text{ kg}}} = 5.33 \ \frac{\text{m}}{\text{s}}$$

This result is rounded to three significant figures as required.

▲

▼ Example 4.7

A baseball batter hits a baseball straight up with a velocity of 180 ft/s. A regulation baseball has a mass of 144.3 g. Ignoring air friction, how high does the baseball go before it comes to a stop?

I work this out using the same method as in the previous problem. First, write down the givens and do the unit conversions.

$$m = 144.3\ \text{g} \cdot \frac{1\ \text{kg}}{1000\ \text{g}} = 0.1443\ \text{kg}$$

$$v = 180\ \frac{\text{ft}}{\text{s}} \cdot \frac{0.3048\ \text{m}}{1\ \text{ft}} = 54.9\ \frac{\text{m}}{\text{s}}$$

$$h = ?$$

Now draw the energy diagram that indicates what is happening in this problem. The kinetic energy the ball has as it leaves the bat converts to gravitational potential energy as it rises, so

$$E_K \rightarrow E_G$$

Since we are starting with kinetic energy this time, compute that first.

$$E_K = \tfrac{1}{2}mv^2 = 0.5 \cdot 0.1443\ \text{kg} \cdot \left(54.9\ \frac{\text{m}}{\text{s}^2}\right)^2 = 217\ \text{J}$$

Since this energy is converting to gravitational potential energy, at the top of the ball's flight we have

$$E_G = 217\ \text{J}$$

Now use this value to solve for the height.

$$E_G = mgh$$

$$h = \frac{E_G}{mg} = \frac{217\ \text{J}}{0.1443\ \text{kg} \cdot 9.80\ \frac{\text{m}}{\text{s}^2}} = 153\ \text{m}$$

Finally, recall that the given velocity has only two significant digits, so we must round this result to two digits, giving

$$h = 150\ \text{m}$$

▲

4.3.5 Energy in the Pendulum

A swinging pendulum provides us with one final example of the conservation of energy in action. To begin, note that because of friction between the swinging pendulum and the air and friction in the pivot at the top, any actual pendulum loses energy to the environment as heat. This is why any actual free-swinging pendulum always comes to a stop.

But let's imagine a perfect pendulum, one that loses no energy due to friction. We call this an *ideal pendulum*. In an ideal pendulum, no energy leaves the "system" (the swinging pendulum) as heat and the pendulum just keeps on swinging without slowing down. (Actually, it's a bit more complicated because of the rotation of the earth, so even in a vacuum with a magnetic bearing at the pivot the pendulum still slows down, so don't start getting visions of a perpetual motion machine! But our imaginary ideal pendulum is also free from such influences.)

From what you know about the forms of energy and energy conservation, you can probably already see how energy transformation works in this ideal pendulum. As shown in Figure 4.6, we let the height of the pendulum when it is at rest (that is, not swinging) be our reference for height measurements. This means when the pendulum is straight down its height is zero and its gravitational potential energy is also zero. When the pendulum swings up to its highest point, it momentarily comes to rest. At this moment, its velocity is zero and so its kinetic energy is zero. Put these facts together and let the pendulum start swinging.

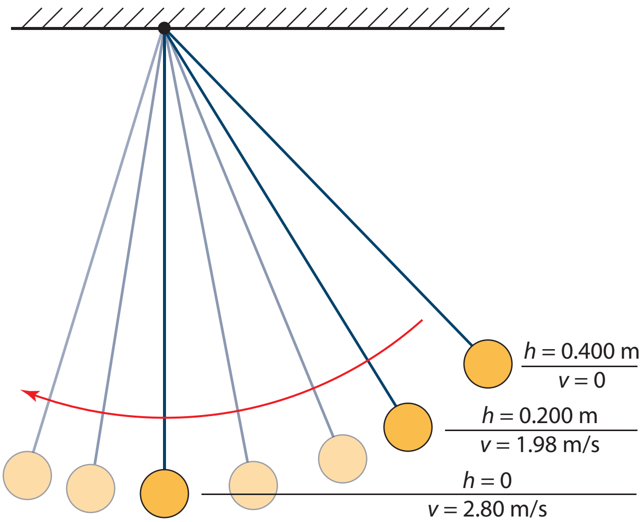


Figure 4.6. Conservation of energy in a swinging pendulum.

Because of the conservation of energy, the pendulum always has the same total amount of energy, no matter where it is. When someone lifts the pendulum to get it started, the person does work on the pendulum equal to the force it takes to lift it times the height (just as we saw with the bucket example). Since the pendulum is ideal, no energy leaves the system as it swings. This means that no matter where the pendulum is, the total energy in the system is always the same and is equal to the amount of energy put into the system in the first place by the work done on it to get it up to where it is released. As the pendulum swings down, E_G converts into E_K, and as the pendulum swings up, E_K converts back into E_G. At all times, the total energy the pendulum possesses always equals the sum of the E_G and the E_K, and this sum always adds up to the same value no matter where the swinging pendulum is.

Just to run through a quick calculation, let's say the mass at the end of the pendulum is 2.00 kg and we lift it up 0.400 m above its lowest point to release it. The total E_G at this starting point is mgh, which gives E_G = 7.84 J. The kinetic energy here is zero, so now we know that the pendulum has a total of 7.84 J of energy no matter where it is.

How fast is it going when it is halfway down, 0.200 m high? Well, the E_G at that position is 3.92 J, so the E_K is 7.84 J – 3.92 J = 3.92 J. Using this kinetic energy value in the velocity equation gives a velocity of 1.98 m/s. At the bottom, all the energy (7.84 J) is kinetic energy, so the velocity is 2.80 m/s. As you see, if you know how high the pendulum is at any point, you can determine how fast it is moving, and vice versa.

Chapter 4 Exercises

Energy Study Questions

1. Write out the stages in the "energy trail" for the following sequences. For each stage, list where the energy is and what form it is in.
   a. From the sun to a ball thrown straight up at the top of its flight.
   b. From the sun to a galloping horse.
   c. From the sun to water stored in a water tower, pumped up there by the city's electric pumps.
   d. From the sun to a moving motorcycle.
   e. From the sun to an electric blow dryer for drying hair.
   f. From the sun to a diesel truck parked at the top of a steep hill.
2. Write down the law of conservation of energy from memory. Then write a paragraph explaining the law in your own words. Include some examples in your explanation.
3. From an energy standpoint, what is an "ideal system"? (Think of the ideal pendulum discussed at the end of the chapter.)
4. If you run out of gas, your car soon comes to a stop. What happens to all the kinetic energy the car has before running out of gas? Where does it go?

Classroom Energy Computation Examples

These examples are written here so that as your teacher works examples in class you can focus on the solutions rather than on worrying about getting the problems written down.

1. Water is pumped into a high water tower. If the total mass of the water is 1.00×10^5 kg and the tower is 240 feet high, what is the gravitational potential energy (E_G) of the water in the tower?
2. A bullet of mass 25 g is fired at a velocity of 556 ft/s. How much kinetic energy (E_K) does the bullet have?
3. A man lifts a bucket of sand 75 cm above the ground. If the bucket of sand has a mass of 12,500 g, how much work does the man do on the bucket?
4. Referring again to the previous problem, after the bucket of sand is lifted, how much E_G does the sand have?
5. If the man releases the bucket of sand and lets it drop, what is its velocity the instant before it strikes the ground?
6. A boy carries a water balloon up to the top of a ladder to let it drop. The mass of the water balloon is 255.8 g and the ladder is 10.4 ft tall.
   a. How much E_G does the balloon have at the top of the ladder?
   b. If the balloon is released, how fast is it going just before it splats on the ground?

Answers

1. 72,000,000 J
2. 360 J
3. 92 J
4. 92 J
5. 3.8 m/s
6. a) 7.95 J; b) 7.88 m/s

Energy Calculations Set 1

1. A load of building materials is hoisted to the top of a building under construction. If the total mass of the material is 1.31×10^3 kg and the building is 177.44 feet high, what is the gravitational potential energy (E_G) of the material at the top of the building?
2. A car of mass 2,345 kg is traveling at a speed of 31 mph. How much kinetic energy (E_K) does the car have?
3. A woman lifts a bucket of water 61.7 cm above the ground. If the bucket of water has a mass of 17.5 kg, how much work does the woman do?
4. How much E_G does the woman's bucket have after being lifted?
5. If the woman releases the bucket of water and lets it drop, what is its velocity the instant before it strikes the ground?
6. A kid shoots an arrow straight up. The arrow has a mass of 122 g and leaves the bow with a velocity of 13.75 m/s. How high does it go above the point where it is shot from the bow?
7. A girl drops a stone into the water from a bridge. The stone has a mass of 325 g and the bridge is 36.1 m above the water. How fast is the stone moving just before it hits the water?
8. A worker slides a carton across the floor. The force of friction between the carton and the floor is 735 N. If the worker pushes the carton 26 m, how much work does he do?

Answers

1. 694,000 J
2. 230,000 J
3. 106 J
4. (This answer is top secret!)
5. 3.48 m/s
6. 9.65 m
7. 26.6 m/s
8. 19,000 J

Energy Calculations Set 2

1. A carpenter hauls 20 bundles of shingles up onto a roof, each bundle weighing 80.0 lb. The roof is 8.5 m above the ground.

a. Compute the total mass of the shingles using the conversion factor for pounds to newtons found in Appendix A, and then the weight equation to get the mass.
b. Compute the work the carpenter has to do to get the shingles up onto the roof. State your answer in joules (J).
c. Using the equation for gravitational potential energy (E_G), compute the E_G of the shingles while they are on the roof.
d. If the entire stack of shingles slides off the roof, how much kinetic energy does the stack have at the following times:
    i. At the moment it first begins to slide.
    ii. Just before it hits the ground.
e. Compute how fast the stack is falling just before it hits the ground.

2. A car weighing 3,193 lb rests at the top of a hill, then begins to roll down the hill. (The engine is off.) Assume it rolls to the bottom of the hill with negligible friction.
    a. If the hill is 16 m high compared to the flat road at the bottom, compute the E_G of the car while it is at the top of the hill.
    b. After the car rolls to the bottom of the hill, where is the energy that is in the E_G of the car while it is at the top of the hill?
    c. Compute the E_K of the car when it reaches the bottom of the hill.
    d. Compute the velocity of the car when it reaches the bottom of the hill.
    e. Explain how conservation of energy relates to this problem.
    f. Explain specifically how friction would change the results of the problem in a more realistic example.

Answers

1. a) 727 kg; b) 6.1×10^4 J; c) 6.1×10^4 J; d) 0 J, 6.1×10^4 J; e) 13 m/s
2. a) 230,000 J; c) 230,000 J; d) 18 m/s

Energy Calculations Set 3

1. Consider a large, ideal (that is, frictionless) pendulum with a steel ball weighing 27.05 lb on the end. Keeping the pendulum cable tight, this ball is lifted to a height of 185 cm and released. Since the pendulum is frictionless, the ball swings back and forth forever.
    a. How much work is done to lift the ball?
    b. What is the E_G of the ball after it is lifted, but before it starts falling?
    c. At the bottom of the ball's pathway, what is its E_G?
    d. At the bottom of the ball's pathway, what is its E_K?
    e. How fast is the ball going at the bottom?
    f. Use friction and energy considerations to explain the difference between this ideal pendulum and an actual pendulum. Which one has the highest velocity at the bottom of its swing?

2. A group of city water pumps pushes water up into a water tower 197 feet high. The water tower holds 6.016×10^6 kg of water. Determine the amount of mechanical work the pumps must do to fill the water tower and state your answer in GJ.

3. Imagine a new frictionless roller coaster that uses magnetic levitation so that the cars float above the rails without actually touching them. Imagine also that the aerodynamic design of the cars is so brilliant that there is essentially no air friction. The car has a mass of 5,122 kg. From the top of a 25.0 m-hill, the car rolls down to a valley where the track is right on the ground. Assuming the roller coaster begins at rest at the top of the hill, determine how fast it is traveling when it reaches the bottom of the valley.
4. A boy weighing 104.6 lb runs up the steps two at a time. There are 13 steps, each one 16.5 cm high.
   a. How much work does he do to get to the top of the steps?
   b. If he steps over the hand rail and drops back down to the ground, how fast is he moving just before he lands?
5. An object with mass m = 351 g is sliding on a frictionless surface at 500.00 cm/s when it begins going up a ramp. What is its height, h, when it stops?

Answers

1. a) 223 J; b) 223 J; c) 0 J; d) 223 J; e) 6.02 m/s
2. 3.54 GJ
3. 22.1 m/s
4. a) 998 J; b) 6.48 m/s
5. 1.28 m

Do You Know ... Why are there pendulums in clocks?

Galileo first discovered that the period of a swinging pendulum depends only on its length (a fact you confirm in the Pendulum Experiment). Because of this, pendulums are used to regulate the motion of the mechanical systems in clocks. The weight on the end of the pendulum is supported by a nut on a threaded rod, and the vertical position of the nut is adjusted to give the pendulum the precise length needed for the clock to run at the correct speed.

The pendulum in a grandfather clock is kept in motion by the gravitational potential energy in the weights hanging inside the cabinet. As the weights slowly descend, their gravitational potential energy is transferred to the energy in the swinging pendulum. The gravitational potential energy in the weights is replenished when a person does work on them, raising them back to their highest position to begin descending again. Because of friction, the pendulum would eventually stop swinging without receiving energy from the continuous action of the descending weights.

As the pendulum swings, it continuously converts its energy from kinetic energy to gravitational potential energy and back again.

APPENDIX C
Laboratory Experiments

C.1 Important Notes

Please refer to pages xii–xiii in the Preface for Teachers for important information pertaining to the terms "experimental error" and "percent difference" as used in this text.

The following pages contain your guidelines for the five laboratory experiments you will conduct in *Introductory Physics* during the year. For each of these experiments, you will submit an individual written report. It is your responsibility to study *The Student Lab Report Handbook* thoroughly so that you can meet the expectations for lab reports in this course.

The instructions written here are given to help you complete your experiment successfully. However, your report must be written in your own words. This applies to all sections of the report. Do not copy the descriptions in this appendix into your report in place of writing your report for yourself in your own words.

C.2 Lab Journals

You must maintain a proper lab journal throughout the year. Your lab journal contributes to your lab grade along with your lab reports. Chapter 1 of *The Student Lab Report Handbook* contains a detailed description of the kind of information you should carefully include in your lab journal entries. The following are highlights from that description.

A good lab journal includes the following features:

1. The pages in the journal are quadrille ruled (graph paper) and the journal entries are in ink.
2. The journal is neatly maintained and free of sloppy marks, doodling, and messiness.
3. Each entry includes the date and the names of the team members present.
4. Every experiment and every demonstration that involves taking data or making observations is documented in the journal.
5. Entries for each experiment or demonstration include:
    - the date
    - the team members' names

- the team's hypothesis
- an accurate list of materials and equipment, including make and model of any electronic equipment or test equipment used
- tables documenting all the data taken during the experiment, including the units of measure and identifying labels for all data
- all support calculations used during the experiment or in preparation of the lab report
- special notes documenting any unusual events or circumstances, such as bad data that require doing any part of the experiment over, unexpected occurrences or failures, or changes to your experimental approach
- little details about the experiment that need to be written in the report that you may forget about later
- important observations or discoveries made during the experiment

C.3 Experiments

Experiment 1 The Pendulum Experiment

Variables and experimental methods

Essential equipment:

- string
- meter stick
- paper clip
- large steel washers
- clock with second hand

This investigation involves a simple pendulum. The experiment is an opportunity for you to learn about conducting an effective experiment. In this investigation, you learn about controlling variables, collecting careful data, and organizing data in tables in your lab journal.

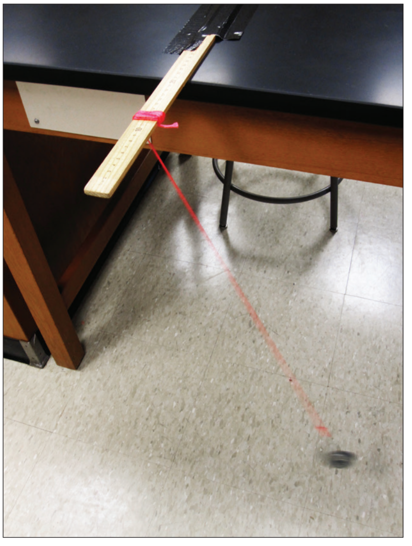
To make your pendulum, bend a large paper clip into a hook. Then connect the hook to a string, and connect the string to the end of a meter stick. Then lay the meter stick on a table with the pendulum hanging over the edge and tape the meter stick down. Finally, hang one or more large metal washers on the hook for the weight.

Your goal in this experiment is to identify the explanatory variables that affect the period of a simple pendulum. A pendulum is an example of a mechanical system that *oscillates*, that is, repeatedly "goes back and forth" in some regular fashion. In the study of any oscillating system, an important parameter is the *period* of the oscillation. The period is the length of time (in seconds) required for the system to complete one full cycle of its oscillation. In this experiment, the period of the pendulum is the response variable you monitor. (Actually, for convenience you monitor a slightly different variable, closely related to the period. This is explained on the next page.) After thinking about the possibilities and forming your team hypothesis, construct your own simple pendulum from string and some weights and conduct tests on it to determine which variables affect its period and which variables do not.

In class, explore the possibilities for variables that may affect the pendulum's period. Within the pendulum system itself there are three candidates, and your instructor will lead the discussion until the class has identified them. (We ignore factors such as air friction and the earth's rotation in this experiment. Just stick to the obvious variables that clearly apply to the problem at hand.)

Then, as a team, continue the work by discussing the problem for a few minutes with your teammates. In this team discussion, form your own team hypothesis stating which variables you think affect the period. To form this hypothesis, you need not actually do any new research or tests. Just use what you know from your own experience to make your best guess.

The central challenge for this experiment is to devise an experimental method that tests only one explanatory variable at a time. Your instructor will help you work this out, but the basic idea is to set up the pendulum so that two variables are held constant while you test the system with large and small values of the third variable to see if this change affects the period. You must test all combinations of holding two variables constant while manipulating the third one. All experimental results must be entered in tables in your lab journal. Recording the data for the different trials requires several separate tables. For each experimental setup, time the pendulum during three separate trials and record the results in your lab journal. Repeating the trials this way enables you to verify that you have valid, consistent data. To make sure you can tell definitively that a given variable is affecting the period, *make the large value of the variable at least three times the small value in your trials.*



Here is bit of advice about how to measure the period of your pendulum. The period of your pendulum is likely to be quite short, only one or two seconds, so measuring it directly with accuracy is difficult. Here is an easy solution: assign one team member to hold the pendulum and release it on a signal. Assign another team member to count the number of swings the pendulum completes, and another member as a timer to watch the second hand on a clock. When the timer announces "GO" the person holding the pendulum releases it, and the swing counter starts counting. After exactly 10.0 seconds, the timer announces "STOP" and the swing counter states the number of swings completed by the pendulum during the trial. Record this value in a table in your lab journal. If you have four team members, the fourth person can be responsible for recording the data during the experiment. After the experiment, the data recorder reads off the data to the other team members as they enter the data in their journals.

This method of counting the number of swings in 10 seconds does not give a direct measurement of the period, but you can see that your swing count works just as well for solving the problem posed by this experiment, and is a lot easier to measure than the period itself. (The actual period is equal to 10 seconds divided by the number of swings that occur in 10 seconds.)

One more thing on measuring your swing count: your swing counter should state the number of swings completed to the nearest 1/4 swing. When the pendulum is straight down, it has either completed 1/4 swing or 3/4 swing. When it stops to reverse course on the side opposite from where it is released, it has completed 1/2 swing.

When you have finished taking data, review the data together as a team. If you did the experiment carefully, your data should clearly indicate which potential explanatory variables affect the period of the pendulum and which ones do not. If your swing counts for different trials of the same setup are not consistent, then something is wrong with your method. Your team must repeat the experiment with greater care so that your swing counts for each different experimental setup are consistent.

Discuss your results with your team members and reach a consensus about the meaning of your data. Expect to spend at least four hours writing, editing, and formatting your report. Lab reports count a significant percentage of your science course grades throughout high school, so you should invest the time now to learn how to prepare a quality report.

Your goal for this report is to begin learning how to write lab reports that meet all the requirements described in *The Student Lab Report Handbook*. One of our major goals for this year is to learn what these requirements are and become proficient at generating solid reports. Nearly all scientific reports involve reporting data, and a key part of this first report is your data tables, which should all be properly labeled and titled.

After completing the experiment, all the information you need to write the report should be in your lab journal. If you properly journal the lab exercise, you will have all the data, your hypothesis, the materials list, your team members' names, the procedural details, and everything else you need to write the report. Your report must be typed and will probably be around two or three pages long. You should format the report as shown in the examples in *The Student Lab Report Handbook*, including major section headings and section content.

Here are a few guidelines to help you get started with your report:

1. There is only a small bit of theory to cover in the Background section, namely, to describe what a pendulum and its period are. You should also explain the experimental method, that is, why we are using the number of swings completed in 10 seconds in our work in place of the actual period. As stated in *The Student Lab Report Handbook*, the Background section must include a brief overview of your experimental method and your team's hypothesis.

2. Begin your Discussion section by describing your data and considering how they relate to your hypothesis. In this experiment, we are not making quantitative predictions, so there are no calculations to perform for the discussion. We are simply seeking to discover which variables affect the period of a pendulum and which do not. Your goal in the Discussion section is to identify what your data say and relate that to your reader.

3. Consider the following questions as you write your discussion. What variables did you manipulate to determine whether they had any effect on the period of the pendulum? What did you find? According to your data, which variables do affect the period? How do the data show this? Refer to specific data tables to explain specifically how the data support your conclusion. Are your findings consistent with your hypothesis? If not, then what conclusion do you reach about the question this experiment seeks to answer?

Experiment 2 The Soul of Motion Experiment

Newton's second law of motion

Essential equipment:

- vehicle
- duct tape
- stop watch
- bathroom weigh scale (2)

Note: The report for this experiment requires you to set up a graph showing predicted and experimental curves on the same set of axes. Procedures for creating such a graph on a PC or Mac are described in detail in *The Student Lab Report Handbook*.

You will have a great time with this experiment. You meet out in the parking lot as a class. The idea is to push a vehicle from the rear, using scales that measure the force the pushers are applying to the vehicle. You time the vehicle as it accelerates from rest through a ten-meter timing zone and use the time data to calculate the experimental values of the vehicle's acceleration. Using the mass of the vehicle and Newton's second law, you calculate a predicted acceleration for each amount of pushing force used. Your goal is to compare your predicted accelerations to the experimental values of acceleration for four different force values. You then graph the results and calculate the percent difference to help you see how they compare.

This experiment is an excellent example of how experiments in physics actually work. The scientists have a theory that enables them to predict, in quantitative terms, the outcome of an experiment. Then the scientists carefully design the experiment to measure the values of these variables and compare them to the predictions, seeking to account for all factors that affect the results. If the theory is sound and the experiment is well done, the results should agree well with the theoretical predictions and the percent difference should be low.

In our case, when a force is applied to a vehicle at rest, we expect the vehicle to accelerate in accordance with Newton's second law of motion:

$$a = \frac{F}{m}$$

This equation predicts that the acceleration depends on the force applied. So Newton's second law is our theoretical model for the motion of an accelerating object. Now, we know that a motor vehicle has a fair amount of friction in the brakes and wheel bearings, which means that not all the force applied by the pushers serves to accelerate the car. Some of it simply overcomes the friction. Also, if the ground is not be perfectly level, this affects the acceleration as well. So to make the model as useful as possible, you must use the actual *net* force on the vehicle in your predictions. Details are discussed below.

For the data collection, you must have a way to measure the actual vehicle's acceleration so that you can compare it to your predictions. You already know an equation that gives the acceleration based on velocities and time. However, you have no convenient way of measuring the vehicle's velocity. (The vehicle moves too slowly for the speedometer to be of any use.) Fortunately, there is another equation you can use if you time the vehicle with a stop watch as it starts from rest and moves through a known distance. If you know the distance and the time, and the acceleration is uniform, you can calculate the vehicle's acceleration as follows:

$$a = \frac{2d}{t^2}$$

Use this equation to determine the experimental acceleration value for each force, using the average time for each set of trials.

Here are some crucial details to help make this experiment as successful as possible:

1. Always have two students pushing on the vehicle. Thus, for each force value the pushers use, the total applied force is twice that amount. (You use four different force values in the experiment.)
2. Measure the friction on the vehicle so you can subtract it from the force the pushers are applying to get the net force applied for your predictions. To measure the friction, use one pusher and estimate the absolute minimum amount of force needed to keep the vehicle barely moving at a constant speed. As you know from our studies of the laws of motion, vehicles move at a constant speed when there is no net force. So if the vehicle is moving at a constant speed, it means that the friction and the applied force are exactly balanced. This allows you to infer what the friction force is.
3. Use four different values of pushing force. For each force value, time the vehicle over the ten-meter timing zone at least three times. The forces the pushers apply to the vehicle always vary quite a bit, so if you get three valid trials at each force you have three reliable data points for the time. You then calculate the average of these times and use it to calculate the experimental value of the acceleration of the vehicle for that force.
4. The major factor introducing error into this experiment is the forces applied by the pushers. Pushing at a constant force while the vehicle is accelerating is basically impossible. (The dial on the force scale jumps all over the place.) But if the pushers are careful, they can push with an *average* force that is pretty consistent. You need a standard to judge whether you have had a successful run with consistent pushing. Here is the criterion to use: when you obtain three trials with time measurements all within a range of one second from highest to lowest, accept those values as valid. If your times are not this close together, assume that the pushing forces are not consistent enough and keep running new trials until you get more consistent data.
5. The instructor will take the vehicle, with a full gas tank, to get it weighed and report this weight to the class. Make sure to measure the weight of the driver and the weight of the scale support rack (if there is one). Add these weights to the weight of the vehicle and determine the mass for this total weight. (Of course, the instructor must also make

sure the gas tank is full on the day of the experiment, since the fuel in the tank typically amounts to 1–2% of the vehicle weight.)

Considerations for Your Report

In the Background section of your report, be sure to give adequate treatment to the theory you are using for this experiment. In the Newton's second law equation, acceleration is directly proportional to force, so a graph of *acceleration* vs. *force* should be linear. In the Background, you should use this concept to explain why you expect your experimental acceleration values to vary in direct proportion to the force. Explain the equations you are using to get the predicted and experimental acceleration values. Since you are using two different equations, your Background section should include explanations for both of them and why they are needed. The force you are using to make your predictions takes friction into account. You need to explain how friction is taken into account, why you are doing so, and how this relates to the equations.

In the Procedure section, don't forget the important details, such as how you measured the friction force, weighed the driver, and judged the validity of your time data.

In the Results section, present all time data in a single table, along with the average times for the trials at each force value applied by the pushers. Present all the predicted values, experimental values, and percent differences (see Preface, pages xviii–xix) in another table or two. Do not forget to state all the other values used in the experiment, such as the vehicle weight, the weights of the driver and support rack, the distance, the total mass you calculate, and the friction force you measure. As *The Student Lab Report Handbook* describes, in any report, all the data collected must be presented, and they all must be placed either in a table or in complete sentences.

In the Discussion, the main feature is a graph of *acceleration* vs. *force*, showing both the predicted and experimental values on the same graph for all four force values. Carefully study Chapter 7 on graphs in *The Student Lab Report Handbook* and make sure your graph meets all the requirements listed.

Variable	Equation	Comments
force	net force = (2 × force for each pusher) – friction force estimate	There are four values of net force, one for each set of trials.
predicted acceleration	predicted accel = (net force)/(total mass)	Net force is as calculated above. Mass is determined from the total weight. There is a predicted acceleration for each value of net force.
experimental acceleration	experimental accel = (2 × distance)/(avg time)^2	Distance is the length of the timing zone. Average time is the average of the three valid times for a given trial. There is an experimental acceleration for each value of net force.

Table C.1. Summary of equations for the calculations.

For your predicted values of acceleration, use the total mass of the vehicle, driver, and support rack. The instructor will tell you the weight of the vehicle, which you record in your lab journal. Also record the weights of the driver and support rack determined during the experiment. Convert the total weight from pounds to newtons, then determine the mass in kilograms by using the weight equation, $F_w = mg$.

For the force values in your predictions, use the nominal amount of force applied (the two pushers' forces combined) less the amount of force necessary to overcome the friction (which is determined during the experiment).

Table C.1 summarizes the calculations you need to perform for each set of trials.

The heart of your discussion is a comparison of the two curves representing acceleration vs. force (displayed on the same graph), and a discussion of how well the actual values of acceleration match up with the predicted values. In addition to this graphical comparison, compare the four predictions to the four experimental acceleration values by calculating the percent difference for each one, presenting these values in a table and discussing them.

To compare the curves, think about the questions below. Do not write your discussion section by simply going down this list and answering each question. (Please spare your instructor the pain of reading such a report!) Instead, use the questions as a guide to the kinds of things you should discuss and then write your own discussion section in your own words. Remember—this is an exercise in learning how to write a well-constructed lab report, not a boring fill-in-the-blank activity.

Thought Questions and Considerations for Discussion

1. Are both the curves linear? What does that mean?
2. Do they both look like direct proportions? What does that imply?
3. Do the curves have similar slopes? What does that imply?
4. How successful are the results? A percent difference of less than 5% for an experiment as crude as this can be considered a definite success. If the difference is greater than 5%, identify and discuss the factors that may have contributed to the difference between prediction and result. In this experiment, there are several such factors, including wind that may have been blowing on the vehicle.
5. Do not make the mistake of merely assuming that the fluctuation in the pushers' forces explains everything without taking into account the precautions you took to eliminate this factor from being a problem (the time data validity requirement).
6. Also do not make the mistake of assuming that friction explains the difference between prediction and result. Friction can only affect the data one way (slowing the vehicle down). So if friction is a factor, the data have to make sense in light of how friction affects the data. But further, since measuring friction and taking it into account in your predictions is part of your procedure, a generic appeal to friction will not do.
7. Finally, do not make the mistake of asserting that errors in the timing or the timing zone distance measurement explain the difference between prediction and result. Consider just how large the percentage error could realistically be in these measurements, and whether that kind of percentage helps at all in explaining the difference you have between prediction and result. For example, the timing zone is 10 m long. If it is carefully laid out on the pavement, it is unlikely that the distance measurement is in error

by more than a centimeter or so. Even including the slight misalignments of the vehicle that crop up, the distance could probably not be off by more than, say, 10 or 20 cm. But this is only 1–2% of 10 m, and if you are trying to explain a percent difference of 5–10% or more this won't do it. Similar considerations apply to the time values. Given the slow speed the vehicle moves, how far off can the timing be? What kind of percentage error would this produce?

Alternate Experimental Method

If your class is using digital devices such as the PASCO Xplorer GLX to read forces, you can use a slightly different experimental method that improves results and lowers the difference between prediction and result. One of the major sources of error in this experiment is the difficulty the pushers have in accurately applying the correct amount of force to the vehicle. If you use bathroom scales to measure the force, there is nothing that can be done about this problem and the pushers simply have to do the best they can.

However, with the digital devices you can eliminate the problem of force accuracy by using the actual average values of the forces applied by the two pushers to calculate the predicted values. The Xplorer GLX can record a data file of the applied force during a given trial, and when reviewing the data file back at your computer you can view the mean value of the force during the trial. You can use this mean value to calculate the predicted acceleration from Newton's second law. Using this method to form your predictions eliminates much of the uncertainty surrounding the forces applied to the car.

Here are a few details to consider if you use this alternative approach to collecting data:

1. You do not need to select four different force values in advance and push the vehicle repeatedly at each force value. Instead, only a single trial is needed for each force.
2. Select 10 or 12 different target force values and run a single trial with each. The force targets should range from low values that barely get the vehicle to accelerate, all the way up to the highest values the pushers can deliver. For each trial, tell the pushers the target force and tell them to do their best to stay on it during the trial. But it doesn't matter nearly so much how accurate the pushers are because you are using the average of the actual data from the digital file to make the predictions, rather than relying on the pushers to maintain the target force accurately.
3. The method for determining values of net force for the predictions is similar to that shown in Table C.1. The difference is that instead of doubling the target force for each pusher, you add together the actual mean forces obtained from the data files for each pusher and subtract out the friction force.
4. Use the time of each trial to determine the experimental value of the acceleration for that trial.
5. Calculate the percent difference for each trial and report these values in the report. Also calculate the average of the percent difference values and use this figure in your discussion of the results.